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Generic element formulation for modelling bolted lap joints

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Abstract

Joints have significant effects on the dynamic response of the assembled structures due to existence of two non-linear mechanisms in their interface, namely slipping and slapping. These mechanisms affect the structural response by adding considerable damping into the structure and lowering the natural frequencies due to the stiffness softening. Neglecting these effects in modelling of joints produces errors in predictions of the structure responses. In this paper, a non-linear generic element formulation is developed for modelling bolted lap joints. The generic element is formed by satisfying all conditions that are known for a joint interface and hence providing a non-linear parametric formulation for the families of allowable joint models. Dynamic response of the developed model for the assembled structure including the generic joint interface element is obtained using the incremental harmonic balance (IHB) method. The generic parameters of the joint are identified by minimising the difference between the model response obtained from IHB method and the observed behaviour of the structure. The procedure is demonstrated by modelling an actual structure containing a single lap bolted joint in the middle. The frequency responses of the structure around the first two resonance frequencies are measured by exciting the structure using a sinusoidal force at each individual frequency. The measured responses are compared with the predictions of the model containing a parametric generic joint element. The parameters of the joint interface model are successfully identified by minimising the difference between the measured responses and the model predictions.
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Keywords: Generic element model; Joint interface modelling; Joint softening effects; Incremental harmonic balance method

1. Introduction

Joints are essential parts of every compound structure and play a significant role in its dynamical response. The added flexibility introduced by the joints heavily affects the structure behaviour and when subjected to dynamic loading, most of energy is dissipated in the joints. Formulating the joint behaviour representing the physical merits involved in the joint interface is critical in modelling any compound structure. The two most common mechanisms involved in the joint interface are frictional slip, and slapping [1]. These mechanisms have their own characteristic features, e.g., frictional slip becomes saturated at very high amplitudes and slapping pushes energy from low-to-high frequencies. The common effects of these mechanisms in the joint behaviour are the stiffness softening, which manifest itself in the decrease of the resonance frequency as the

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amplitude of excitation load increases, jump phenomenon in the frequency response curve and dissipation of energy, which significantly reduces the vibration amplitude at resonance frequency.

There are two common approaches used in modelling the joint properties. The first approach employs non-parametric models and is widely used since no assumption about the properties of the joint is required. In this approach, the joint flexibility and damping effects are modelled using added terms such as an equivalent excitation force which produces the same effects as a joint in the dynamic behaviour of the structure [2–5]. The second approach in modelling joints employs parametric models. In contrast with non-parametric models, in using the parametric models it is necessary to have a good understanding about the involved physics of a joint. Common phenomena in the joint interface are micro/macro slips/slaps that cause energy dissipation and non-linearity in the joints. Most parametric models are proposed to represent the non-linearity and energy dissipation due to slip in the joints. Reviews on joint friction models are presented in Refs. [6–12].

The parametric models of joint interfaces are categorised into the zero thickness elements and the thin layer element theories. Both of these two categories are initially developed in geomechanics [13,14] and later were used in other engineering fields. In the zero thickness elements, sometimes referred to lumped models, the effect of joint is considered to be concentrated at a single point and no dimensions is considered for the joint. Examples of zero thickness elements are lumped springs, and frictional sliders which are commonly used to represent the force–displacement relationship in the joint interface. Several linear and non-linear lumped joint models have been proposed. A widely used model to represent the stick–slip response of the jointed structures is proposed by Iwan [9]. Iwan used a combination of parallel/series spring-slider elements to describe the hysteresis behaviour of materials and structures. Gaul and Lenze [11] used the Valanis model to simulating the stick–slip in the joint interface of the assembled structures. Canudas de Wit et al. [15] proposed a six-parameter model which is capable to represent the response of joints with microslip. Segalman et al. [1] proposed the so-called *Hysteresis Model* which represents a power law relationship between the force and displacement at the joint interface. Segalman [16] showed that Iwan model can be successfully used for representing the micro/macro slip at joint interfaces. Menq et al. [17,18] developed a one-dimensional, physically motivated micro-slip model that allows partial slip on the friction interface. Ahmadian and Jalali [19] proposed a non-linear joint model composed of a set of linear and non-linear springs capable of representing the softening effect and jump phenomenon in the joint interface. They used an equivalent linear viscous damping to model the dissipation energy in the joint interface.

In the second category of joint models called thin layer element theory the joint is represented as an element with physical dimensions, and specific force–displacement relationship. Ahmadian et al. [20–22] in a series of papers showed that the linear thin layer elements can be used successfully in modelling the linear behaviour of the various types of joints such as bolted, spot welded, and segment to segment contact joints. Song et al. [10] proposed an adjusted Iwan beam element. The proposed joint element uses an Iwan model in parallel with a spring to formulate the non-linear force–displacement model both in shear and bending deformations.

This paper presents a method for development of joint interface elements based on the thin layer element approach. The method forms a generic element model [23,24] capable of defining families of the allowable models for the joint interface under consideration. Although a generic beam element is presented in this paper, the method is general and can be extended to develop other elements such as generic solid elements to model the joint interfaces between plates or bulk solid parts. In formulating a generic joint interface element, geometry of the element is selected and a parametric model is formed that satisfies all known properties of the element such as the definiteness properties of its matrices, the matrices null spaces, any constraints due to geometrical symmetry of the element, etc. The result is a generic model capable of presenting families of acceptable models for the joint interface. The unknown parameters of the generic joint interface element are identified by comparing the response of the developed model with corresponding experimental observations.

A generic joint element formulation for bolted single lap joints consists of a non-linear generic stiffness, a generic damping matrix, and a generic mass matrix is presented to demonstrate the procedure involved in development of non-linear a generic joint model. The stiffness non-linearity is assumed to be of cubic form. The generic model is capable of representing the softening effects and the damping in the joints interface. The proposed generic formulation is used in modelling an assembled beam like structure containing a single lap bolted joint in the middle. The model frequency responses are derived using incremental harmonic balance

(IHB) method [25]. The model predictions are then compared with the experimental measured frequency responses of the structure to identify the generic parameters of the joint interface model.

The paper is organised as following. Section 2 formulates the joint interface generic element model capable of presenting the behaviour of a single lap bolted joint. The response of the assembled structure modelled using the generic element formulation is obtained using the method of IHB as presented in Section 3. The obtained analytical solution for the problem is a great advantage in the parameter identification procedure as repeated evaluation of the response is required in this process. The generic parameters of the joints are identified in Section 4. The identified model presents good agreements with the experimental observations indicating the joint interface model accurately captures the dominant physics involved in the joint interface.

2. The joint generic element model

Fig. 1 shows a structure consists of two beams connected by a bolted single lap joint. A finite element model representation of the structure is also shown in this figure. The beam parts of structure are modelled using two dimensional (2-D) Euler–Bernoulli beam elements and the joint interface is modelled using a 2 noded/4 dof generic element. The generic joint element model is consisting of parametric stiffness, mass, and damping matrices which form the family of admissible models for the bolted lap joint.

The generic element degrees of freedom are consistent with those of the beam element, i.e., $U=[w_i,L\theta_{\mathrm{i}},w_{i+1},L\theta_{i+1}]^{\mathrm{T}}$, as shown in Fig. 2. The rotational degrees of freedom are multiplied by the length of the element, L to simplify the element formulation.

The joint is assumed to be symmetric with respect to its centre axis. This is a valid assumption as the nodal forces can be moved along the sides of the element with no effect on the force–displacement relations. As will be seen later in the paper the geometrical symmetry reduces the number of model parameters and hence simplifies the modelling.

A generic element model [23,24] is developed by imposing all necessary conditions that model matrices must satisfy; the mass, stiffness, and damping matrices are symmetric, the mass matrix is positive definite, stiffness and damping matrices are positive semi-definite, the element rigid body modes form the null space of the stiffness and damping matrices, the total mass and moments of inertia of the element are obtained by transforming the mass matrix to the modal coordinates using the rigid-body modes. Moreover, if the element

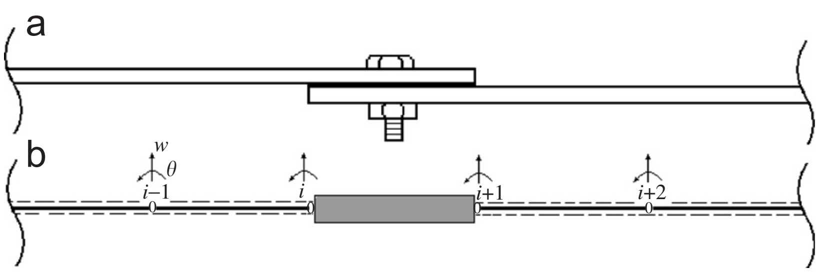

Fig. 1. Modelling of a single lap bolted joint.

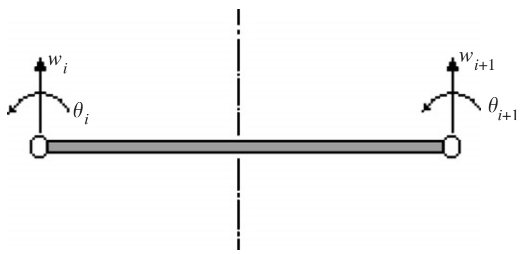

Fig. 2. The joint element degrees of freedom.

has any geometrical symmetry, an out-of-plane rotation of the element about the symmetry axis by 180° does not change the entries of its matrices. The following describes a procedure for constructing generic stiffness, mass, and damping matrices for the bolted lap joints.

2.1. The generic stiffness matrix

The joint element has 4dofs and in general its stiffness matrix can be written in the following form:

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ & k_{22} & k_{23} & k_{24} \\ & & k_{33} & k_{34} \\ Sym. & & & k_{44} \end{bmatrix}. \quad (1)$$

The joint element shown in Fig. 2 has a symmetry axis and therefore an out-of-plane rotation of the element about the symmetry axis by 180° does not change its stiffness matrix, i.e.:

$$T^{\mathrm{T}}KT = K, \quad T = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}. \quad (2)$$

Rotating the element using the transformation matrix T and equating the results to the stiffness matrix in its original form (before transformation), one may reduce the number of independent parameters in the stiffness matrix to six:

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ & k_{22} & -k_{14} & k_{24} \\ & & k_{11} & -k_{12} \\ Sym. & & & k_{22} \end{bmatrix}. \quad (3)$$

The element rigid body modes span the null space of the stiffness matrix, that is

$$K\Phi = 0, \quad \Phi = \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1/2 & 1 & 1/2 & 1 \end{bmatrix}^{\mathrm{T}}. \quad (4)$$

Imposing the above conditions, one may define the generic joint element stiffness matrix using only two independent parameters as

$$K_e = \begin{bmatrix} k_{11} & \frac{k_{11}}{2} & -k_{11} & \frac{k_{11}}{2} \\ & k_{22} & -\frac{k_{11}}{2} & \frac{k_{11}}{2} - k_{22} \\ & & k_{11} & -\frac{k_{11}}{2} \\ Sym. & & & k_{22} \end{bmatrix}. \quad (5)$$

The parameters k_{11} and k_{22} are the lateral and the flexural stiffnesses of the joint element, respectively. The definiteness property of the stiffness matrix requires $4k_{22}>k_{11}>0$. One may simplify the definiteness requirements by defining $k_w = k_{11}$ and $k_\theta = k_{22} - k_{11}/4$; which leads to $k_\theta > 0$, $k_w > 0$. The linear force–displacement relationship in a joint element generic stiffness matrix is expressed as

$$\begin{Bmatrix} V_i \\ M_i/L \\ V_{i+1} \\ M_{i+1}/L \end{Bmatrix} = \begin{bmatrix} k_w & \frac{k_w}{2} & -k_w & \frac{k_w}{2} \\ & k_\theta + \frac{k_w}{4} & -\frac{k_w}{2} & -k_\theta + \frac{k_w}{4} \\ & & k_w & -\frac{k_w}{2} \\ Sym. & & & k_\theta + \frac{k_w}{4} \end{bmatrix} \begin{Bmatrix} w_i \\ L\theta_i \\ w_{i+1} \\ L\theta_{i+1} \end{Bmatrix}, \quad k_w > 0, \quad k_\theta > 0 \tag{6}$$

where $V_i, V_{i+1}, M_i, M_{i+1}$ represent the shear forces, and the bending moments, in nodes i and i+1, respectively. The generic stiffness matrix given in Eq. (6) forms the Euler–Bernoulli beam stiffness matrix when the parameters are set to $k_w = 12EI/L^3$, $k_\theta = EI/L^3$. The Timoshenko beam stiffness matrix can also be obtained by setting: $k_w = 12EI/(1+g)L^3$, and $k_\theta = EI/L^3$ where g is the ratio between shear and bending properties. The force–displacement relations in the linear joint model are derived as

$$\begin{Bmatrix} V_i \\ M_i/L \\ V_{i+1} \\ M_{i+1}/L \end{Bmatrix} = \begin{Bmatrix} k_w \Delta w \\ \frac{1}{2} k_w \Delta w + k_\theta \Delta\theta \\ -k_w \Delta w \\ \frac{1}{2} k_w \Delta w - k_\theta \Delta\theta \end{Bmatrix}, \quad \Delta w = w_i - w_{i+1} + \frac{L}{2}(\theta_i + \theta_{i+1}), \quad \Delta\theta = L(\theta_i - \theta_{i+1}). \tag{7}$$

This linear relationship is valid for infinitesimal deformations produced in the joint. As the amplitude of the excitations exceeds a particular value, the deformations become finite, creating slippage and slapping in the joint. The non-linearities in the joint interface are directly related to the force–displacement relations. In the other words, in a non-linear joint k_w and k_θ are functions of $\{\Delta w, \Delta\theta\}$ and in effect the bending moments and shear forces are non-linear functions of the displacements, i.e.,

$$\begin{Bmatrix} V_i \\ M_i/L \\ V_{i+1} \\ M_{i+1}/L \end{Bmatrix} = \begin{Bmatrix} f_1(\Delta w) \\ \frac{1}{2} f_1(\Delta w) + f_2(\Delta\theta) \\ -f_1(\Delta w) \\ \frac{1}{2} f_1(\Delta w) - f_2(\Delta\theta) \end{Bmatrix}. \tag{8}$$

In constructing the non-linear generic stiffness matrix one must select admissible constitutive functions $f_1(\Delta w)$, $f_2(\Delta\theta)$ to define the force–displacement relations. An admissible choice for the constitutive function is a single Jenkins element consists of a linear/rotational spring (of stiffness k_w/k_θ) and a frictional damper (with sliding force/moment of F/T) in series. Corresponding force–displacement functions are [7]

$$f_1(\Delta w) = \begin{cases} f(t), & |f(t)| < F \\ F\,sign(\Delta\dot{w}) & else \end{cases} \qquad f_2(\Delta\theta) = \begin{cases} \tau(t), & |\tau(t)| < T \\ T\,sign(\Delta\dot{\theta}) & else \end{cases}.$$
$$f(t) = k_w(\Delta w - \Delta w_{rev}) + F\,sign(\Delta\dot{w}_{rev}) \qquad \tau(t) = k_\theta(\Delta\theta - \Delta\theta_{rev}) + F\,sign(\Delta\dot{\theta}_{rev}) \tag{9}$$

where $\Delta w_{rev}/\Delta\theta_{rev}$ is the displacement/rotation immediately prior to velocity reversal. Such a model represents the stick–slip behaviour in the joint but is not capable of capturing the micro-slip effects. The Iwan model [9], which is a combination of Jenkins elements in parallel, is a more representative choice for the joint force–displacement constitutive relations capable of defining both micro and macro slips in the joint. Song et al. [10] developed adjusted Iwan beam model to represent jointed structures by employing the Iwan model in force–displacement/moment–rotation relations of a beam element.

In this study, non-linear elastic stiffness models are employed in defining $f_1(\Delta w)$,$f_2(\Delta\theta)$ and the dissipation energy mechanisms are represented using separate rate-dependant functions. The adopted stiffness functions are

$$f_1(\Delta w) = k_w \Delta w - k_{wN} \Delta w^3, f_2(\Delta\theta) = k_\theta \Delta\theta - k_{\theta N} \Delta\theta^3. \tag{10}$$

The selected polynomial forms pave the way for obtaining the frequency responses directly using analytical methods; a great advantage in parameter identification procedures in which repeated frequency response analysis and comparison with measured data is required. Assigning a negative sign for the non-linear term results softening effects in the model which is consistent with the physics involved in the joint interface problem. The power of non-linear term is selected equal to 3 based on the observations that usually a 3rd super-harmonics appears in the response of the system [26]. The constitutive relations in the non-linear generic joint stiffness model are defined using four unknown parameters namely $k_w, k_{wN}, k_\theta, k_{\theta N}$. These parameters are identified by minimising the difference between the model response and the experimental observations.

2.2. The generic damping matrix

The major source of damping in bolted lap joints is friction. The simplest model to represent the friction force is Coulomb friction model categorised as a static friction model. Gaul and Nitsche [7] suggest adding viscous friction in modelling joint friction to include dynamic effects based on the observations that in sliding the friction force often depends on relative sliding velocity at the friction interface (Stribeck effect). An example of dynamic friction models is LuGre model [27] which defines the friction force as a function of sliding velocity. A velocity-dependant friction model is adopted in this paper to capture the dynamic effects in friction mechanism; it also allows analytical studies which are generally not possible with discontinuous static friction models.

The general form for the damping matrix of the generic joint element can be written as

$$C_e = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ & c_{22} & c_{23} & c_{24} \\ & & c_{33} & c_{34} \\ Sym. & & & c_{44} \end{bmatrix}. \tag{11}$$

Based on the geometric symmetry properties of the element and the null space of the damping matrix, one may define the generic damping model using only two independent parameters:

$$C_e = \begin{bmatrix} c_w & \frac{c_w}{2} & -c_w & \frac{c_w}{2} \\ & c_\theta + \frac{c_w}{4} & -\frac{c_w}{2} & -c_\theta + \frac{c_w}{4} \\ & & c_w & -\frac{c_w}{2} \\ Sym. & & & c_\theta + \frac{c_w}{4} \end{bmatrix}, c_w = c_{11}, \quad c_\theta = c_{22} - \frac{c_{11}}{4}, \quad c_w > 0, \quad c_\theta > 0. \tag{12}$$

A first-order approximate of a rate-dependant non-linear damping behaviour is an equivalent linear viscous damping that approximates the non-linear behaviour in anticipated applied loads. The equivalent damping coefficients are obtained by performing experiments using excitation forces comparable to those of the service loads. For the linear viscous damping model the force–velocity relations are defined as

$$\begin{Bmatrix} V_i \\ M_i/L \\ V_{i+1} \\ M_{i+1}/L \end{Bmatrix} = \begin{Bmatrix} c_w \Delta\dot{w} \\ \frac{1}{2} c_w \Delta\dot{w} + c_\theta \Delta\dot{\theta} \\ -c_w \Delta\dot{w} \\ \frac{1}{2} c_w \Delta\dot{w} - c_\theta \Delta\dot{\theta} \end{Bmatrix}. \tag{13}$$

The constitutive relations in the equivalent viscous damping model are defined using two unknown parameters namely c_w, and c_θ. These parameters can be identified by minimising the difference between the model response and the experimental observations. The generic damping model can be extended to a non-linear form by defining non-linear relations between the forces and velocities:

$$\begin{Bmatrix} V_i \\ M_i/L \\ V_{i+1} \\ M_{i+1}/L \end{Bmatrix} = \begin{Bmatrix} g_1(\Delta\dot{w}) \\ \frac{1}{2}g_1(\Delta\dot{w}) + g_2(\Delta\dot{\theta}) \\ -g_1(\Delta\dot{w}) \\ \frac{1}{2}g_1(\Delta\dot{w}) - g_2(\Delta\dot{\theta}) \end{Bmatrix}. \tag{14}$$

The functions $g_1(\Delta\dot{w}), g_2(\Delta\dot{\theta})$ are admissible non-linear damping model and can be selected as those such as LuGre model [27]. Another approach in defining these functions is to perform a number of tests of increasingly large amplitudes on the jointed structure, and to identify functions $g_1(\Delta\dot{w}), g_2(\Delta\dot{\theta})$ for the joint associated with each load level. The obtained model would demonstrate a power law relationship between the energy loss per cycle and the input force for a given joint preload as shown in [28].

2.3. The generic mass matrix

The general form for the mass matrix of the generic joint element can be written as a 4×4 matrix. Rotating the mass matrix using the transformation matrix T given in Eq. (2) should not make any changes in its entries. Using this symmetry property one may reduce the number of independent parameters in the mass matrix to six:

$$M_e = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ & m_{22} & -m_{14} & m_{24} \\ & & m_{11} & -m_{12} \\ Sym. & & & m_{22} \end{bmatrix}. \tag{15}$$

The element mass and moment of inertia can be related to the model by transforming the mass matrix into modal domain using element rigid-body modes. This results into two more constraints on the entries of the mass matrix:

$$\Phi^T M_e \Phi = \begin{bmatrix} m & 0 \\ 0 & \dfrac{J}{L^2} \end{bmatrix} \Rightarrow \begin{cases} m_{11} + m_{13} = \dfrac{m}{2} \\ \dfrac{m_{11} - m_{13}}{2} + 2 \times (m_{22} + m_{24} - m_{12} - m_{14}) = \dfrac{J}{L^2} \end{cases} \tag{16}$$

where m and J are the combined mass and moment of inertia of the jointed parts of the beams, the bolt, and the nut. These two more constraints reduce the number of independent parameters to 4. Stavrinidis et al. [29] showed that the remaining four parameters affect the discretisation error in a beam element formulation. They obtained the 4 parameters corresponding to a beam element by minimising the FE model discretisation error. In this study, the discretisation error of the joint element is not a prime concern and to present a simple formulation a block diagonal form for the mass matrix is selected. This results the following positive-definite mass matrix for a symmetric joint element:

$$M_e = \frac{m}{2} \begin{bmatrix} 1 & \frac{1}{2} & 0 & 0 \\ & R^2 + \frac{1}{4} & 0 & 0 \\ & & 1 & -\frac{1}{2} \\ Sym. & & & R^2 + \frac{1}{4} \end{bmatrix}, \quad R^2 = \frac{J}{mL^2}. \tag{17}$$

In the following section dynamic response of the jointed structure modelled by the generic mass, stiffness and damping models is obtained using the IHB method.

3. Qualitative solution of the model

The jointed beam-like structure that its behaviour is under investigation is shown in Fig. 3. The structure consists of two steel beams of equal length joined together using a bolted single lap joint. A tip mass is placed at one of the free ends of the assembled structure in order to remove the symmetry axis of the structure. This moves the modal nodes away from the joint location and both slapping and slipping accrue in the joint at low frequencies. Depending on the preload of the bolted joint and the level of excitation force, it is possible to produce variety of structural behaviours ranging from linear, weakly non-linear up to strongly non-linear responses and chaotic motions.

A finite element model of the assembled structure is developed by modelling each beam portion using twelve Euler–Bernoulli beam elements, a generic joint element to model the lap joint part and a lumped mass model for the tip mass. The mass matrix of the tip mass is defined as

$$M_t = \begin{bmatrix} m_t & \\ & I_t \end{bmatrix} \tag{18}$$

where m_t and I_t are the mass and the moment of inertia of the tip mass, respectively.

The generic element model representing the bolted lap joint consists of a generic stiffness matrix with linear and cubic stiffness terms as defined in Eqs. (8) and (10), and an equivalent linear viscous damping matrix as represented in Eq. (13). The 6 unknown parameters in the joint model i.e., k_w,k_{wN},k_θ,$k_{\theta N}$, c_w, and c_θ are identified by minimising the difference between the model predictions and corresponding experimental observations. The mass distribution of the joint element is known and is represented by the block diagonal mass matrix defined in Eq. (17). This matrix models the distribution of the mass and moment of inertia of overlapped parts of the beams, the bolt, and the nut. Fig. 4 shows a schematic of the finite element model of the structure.

The equations of motion of the assembled structure are of the following form:

$$M\frac{d^2q}{dt^2} + C\frac{dq}{dt} + [K_L + K_{NL}(q)]q = f(t) \tag{19}$$

where M is the structure mass matrix, C the structure damping matrix, K_L the linear part and K_{NL} the non-linear part of the structure stiffness matrix, q is the vector of displacements and $f(t)$ is the vector of applied external forces.

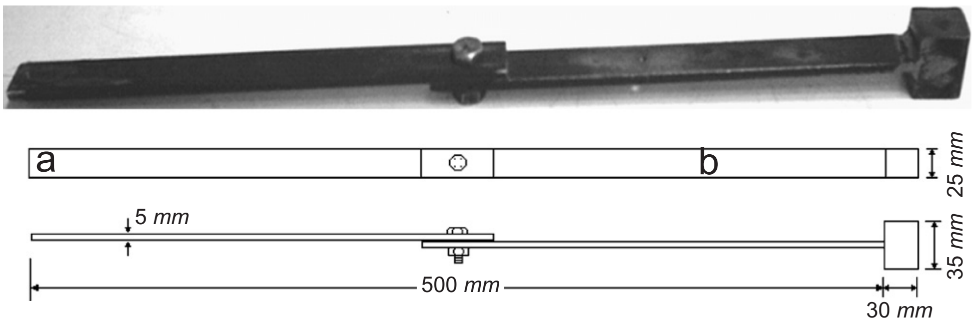

Fig. 3. The assembled structure.

Fig. 4. Finite element model of the assembled structure.

The goal is to solve Eq. (19) and to obtain the frequency response curves of the structure directly. The IHB method is adopted here to solve the problem as it directly leads to the frequency response curves of the structure. It is assumed that the excitation force is a simple harmonic loading with the excitation frequency ω. Introducing a new variable $\tau = \omega t$ into Eq. (19) results

$$\omega^2 M\ddot{q} + \omega C\dot{q} + [K_L + K_{NL}(q)]q = f\ \sin(\tau) \tag{20}$$

where the dots represent differentiation with respect to the variable τ. Let q_0 represent the vibration amplitudes at ω_0. A small increment in the vibration frequency leads to the corresponding increments in the vibration amplitudes:

$$\omega = \omega_0 + \Delta\omega \quad \Rightarrow \quad q = q_0 + \Delta q. \tag{21}$$

Substituting Eq. (21) into Eq. (20) and considering only linear terms while neglecting second and higher-order perturbation terms, and assuming a cubic non-linearity form in the stiffness matrix, one obtains the following linearised incremental equation:

$$\begin{aligned}&\omega_0^2 M\Delta\ddot{q} + \omega_0 C\Delta\dot{q} + [K_L + 3K_{NL}]\Delta q = R - (2\omega_0 M\ddot{q}_0 + C\dot{q}_0)\Delta\omega\\ &R = f\sin(\tau) - (\omega_0^2 M\ddot{q}_0 + \omega_0 C\dot{q}_0 + [K_L + K_{NL}]q_0).\end{aligned} \tag{22}$$

The corrective vector R vanishes when the solution is obtained. The stiffness matrix has a cubic non-linearity and therefore the odd numbered harmonies of the excitation frequency exist in the steady-state response:

$$\begin{aligned}q_0 &= \sum_{k=1}^{Nc}\{a\}_k\ \sin(2k-1)\tau + \{b\}_k\ \cos(2k-1)\tau\\ \Delta q &= \sum_{k=1}^{Nc}\{\Delta a\}_k\ \sin(2k-1)\tau + \{\Delta b\}_k\ \cos(2k-1)\tau.\end{aligned} \tag{23a}$$

The series representations in Eq. (23a) can be rewritten in matrix notation as

$$\begin{aligned}&q_0 = SA, \quad \Delta q = S\Delta A,\\ &A = \begin{Bmatrix}\{a\}_1\\ \{b\}_1\\ \{a\}_2\\ \{b\}_2\\ \vdots\end{Bmatrix}, \quad S = \begin{bmatrix}C_s & & & 0\\ & C_s & & \\ & & \ddots & \\ 0 & & & C_s\end{bmatrix}\\ &C_s = [\sin\ \ \tau, \ \ \sin\ \ 3\tau, \ \ \ldots, \ \ \cos\ \ \tau, \ \ \cos\ \ 3\tau, \ldots\ \ ].\end{aligned} \tag{23b}$$

The unknown response A, and its corresponding increments in the vibration amplitudes ΔA, are determined by minimising the residues of Eq. (22) using the Galerkin method. Substituting the solution form of Eq. (23b) into Eq. (24), pre-multiplying both sides of the resultant equation by S^{T}, and integrating over the range of $\tau = 0.2\pi$ produces

$$\begin{aligned}&(\omega_0^2\overline{M}, +\omega_0\overline{C} + \overline{K} + 3\overline{K}_N)\Delta A = \overline{R} - \Delta\omega(2\omega_0\overline{M} + \overline{C})A\\ &\overline{R} = F - (\omega_0^2\overline{M} + \omega_0\overline{C} + \overline{K} + \overline{K}_N)A\\ &\overline{M} = \int_0^{2\pi} S^{\mathrm{T}}M\ddot{S}\,\mathrm{d}\tau, \quad \overline{C} = \int_0^{2\pi} S^{\mathrm{T}}C\dot{S}\,\mathrm{d}\tau, \quad \overline{K} = \int_0^{2\pi} S^{\mathrm{T}}KS\,\mathrm{d}\tau\\ &\overline{K}_N = \int_0^{2\pi} S^{\mathrm{T}}K_{NL}S\,\mathrm{d}\tau, \quad F = \int_0^{2\pi} S^{\mathrm{T}}f\ \sin\tau\,\mathrm{d}\tau.\end{aligned} \tag{24}$$

Eq. (24) can be solved using an iterative solution procedure starting with the linear response as the initial. The non-linear amplitude frequency response is obtained at each required frequency by incrementing frequency ω or one component of the amplitude vector A. Ref. [30] describes details of the solution procedure and uses the incremental arc-length method to overcome the difficulties in obtaining the solution around sharp response peaks.

4. Experimental case study

In order to validate the proposed generic element model an experimental case study is conducted. Fig. 5 shows the test set-up for measuring the dynamic response of the assembled structure containing a bolted lap joint in the middle. Each part of the structure is suspended as a bifilar pendulum to simulate the free–free boundary condition in lateral vibrations. A *B&K 4810 mini-shaker* is used to excite the structure at one end (point A). The applied force is measured using a *B&K 8100* force transducer and the response at point B is measured by a *DJB A/120/V* accelerometer. Points A and B are shown in Fig. 3. The tests are conducted in two steps; first the structure is excited using a low amplitude pseudo-random force to record its linear behaviour. Next, the structure is excited using a higher level sinusoidal force at selected frequencies near its resonances to capture the non-linear behaviour. The bolt preload remains constant during all experiments.

The linear parameters of the structure k_w and k_θ are obtained using an inverse eigen-sensitivity approach. The non-linear stiffness parameters, k_{wN} and $k_{\theta N}$, and the damping coefficients, c_w and c_θ, are identified by minimising the difference between the measured non-linear response and the response predicted by the model. The model non-linear responses are produced using IHB method. In the IHB solution of the problem only the prime harmonics are considered as no other harmonics were observed in the measurements. The followings describe the experimental results and the identification procedures of linear and non-linear generic joint element parameters.

4.1. Identification of liner joint model

In the first step of the test series a pseudo-random excitation with low amplitude is conducted to measure the linear frequency response of the assembled structure. Fig. 6 shows the frequency response curve of the structure at point B when it is excited at point A.

The frequency response curve shows symmetry in the vicinity of the resonance frequencies and low damping which are typical properties of a linear structure response. The first three natural frequencies are measured to be used in identifying the linear parameters of the joint model.

The proposed joint model should be able to predict the experimental observations presented in Fig. 6. The joint model has six unknown parameters to be identified using the experimental results, namely $k_w, k_\theta, k_{wN}, k_{\theta N}, c_w, c_\theta$. The softening effects represented by $k_{wN}, k_{\theta N}$ and damping in the model are contributed by the non-linear mechanisms in the joints. Therefore, there remains two parameters k_w and k_θ, to be identified using the three measured natural frequencies of the linear system. The identification is performed using the linearised eigen-sensitivity method with the initial values set as those of a beam element, i.e., $k_{w0} = 12EI/L^3$

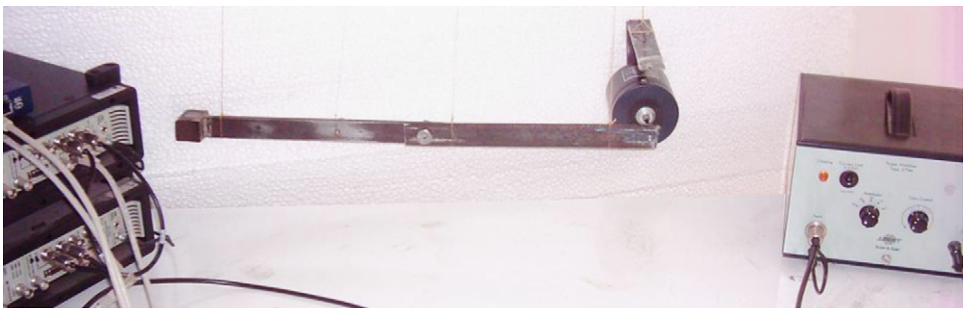

Fig. 5. The test set-up.

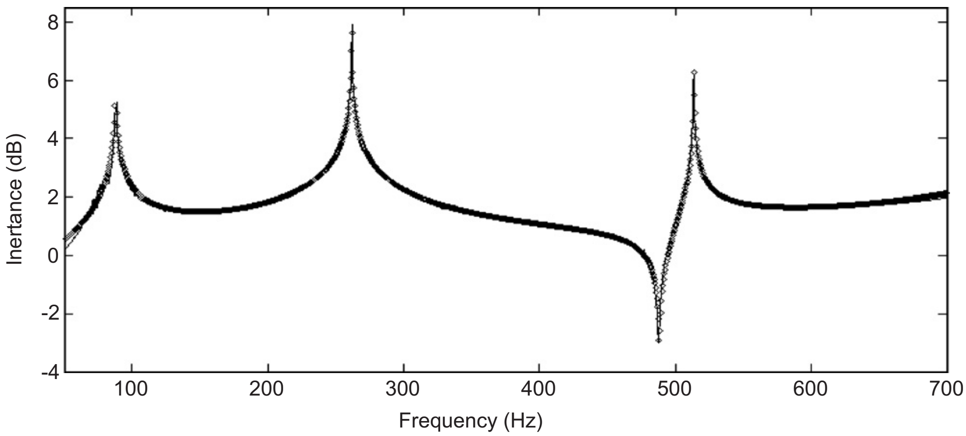

Fig. 6. FRF of the assembled structure, experiment (solid line) model (dots).

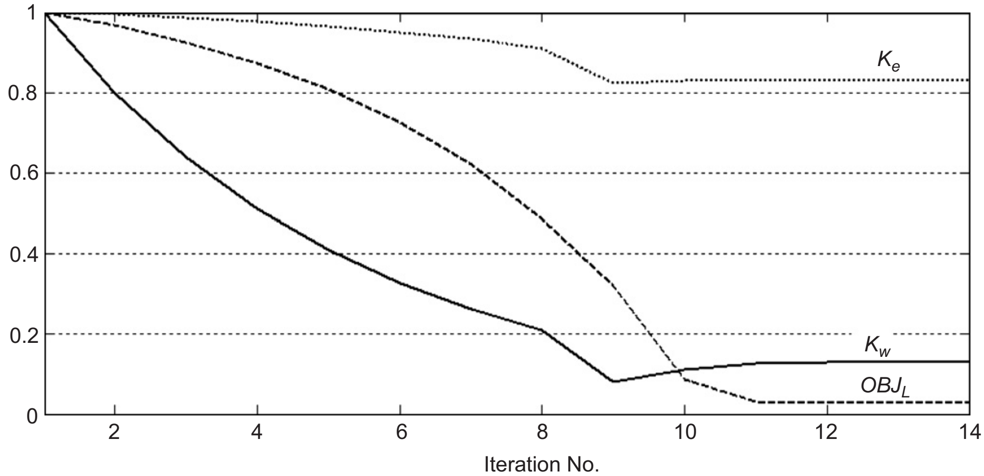

Fig. 7. Changes in objective function and the joint parameters during identification process.

and $k_{\theta 0} = EI/L^3$. Note that the stiffness of joint in the initial model is 1/8 of the corresponding welded lap joint. The objective function is formed using the norm of the difference between measured eigen-values and the eigen-values predicted by the model. The change in objective function from its initial value during the identification procedure is shown in Fig. 7. The objective function reduces to 3% of its initial value which indicates the success of identification procedure and the effectiveness of selected parameters in modelling the linear joint. The corresponding changes in the joint parameters k_w and k_θ are also shown in Fig. 7 which indicate 17% reduction in k_θ and 87% reduction in k_w from their initial values. The stiffness parameters are reduced from the initial values during the identification procedure due to flexibility of the bolted joint.

Table 1 tabulates the measured natural frequencies and the identified model predictions of these frequencies. The results show very good agreement with the observed behaviour. The predicted frequency response of the identified model which is overlaying the measured one in Fig. 6 confirms the good correlations between the predictions of the identified model and the experimental measurements.

The change in parameter k_w is almost one order of magnitude higher than the change in k_θ. This can be justified by considering the sensitivity of initial model eigen-values to the changes of k_θ is one order of magnitude higher than the corresponding values of k_w. The sensitivity coefficients are tabulated in Table 2. Table 2 also shows these sensitivity coefficients when the tip mass is removed from the model. Comparing the

Table 1
Measured and predicted natural frequencies

	First mode (Hz)	Second mode (Hz)	Third mode (Hz)
Measured	88.18	262.0	513.50
Predicted	88.15	261.82	513.84
Error (%)	−0.035	−0.031	0.067

Table 2
The sensitivity of natural frequencies respect to joint parameters

	$\frac{\partial \omega_j^2}{\partial k_w} k_w$		$\frac{\partial \omega_j^2}{\partial k_\theta} k_\theta$	
	Without tip mass	With tip mass	Without tip mass	With tip mass
First mode	0.0	0.135×10^3	1.158×10^5	60.344×10^3
Second mode	1.541×10^5	70.505×10^3	0.0	599.590×10^3
Third mode	0.0	126.860×10^3	24.033×10^5	1184.200×10^3

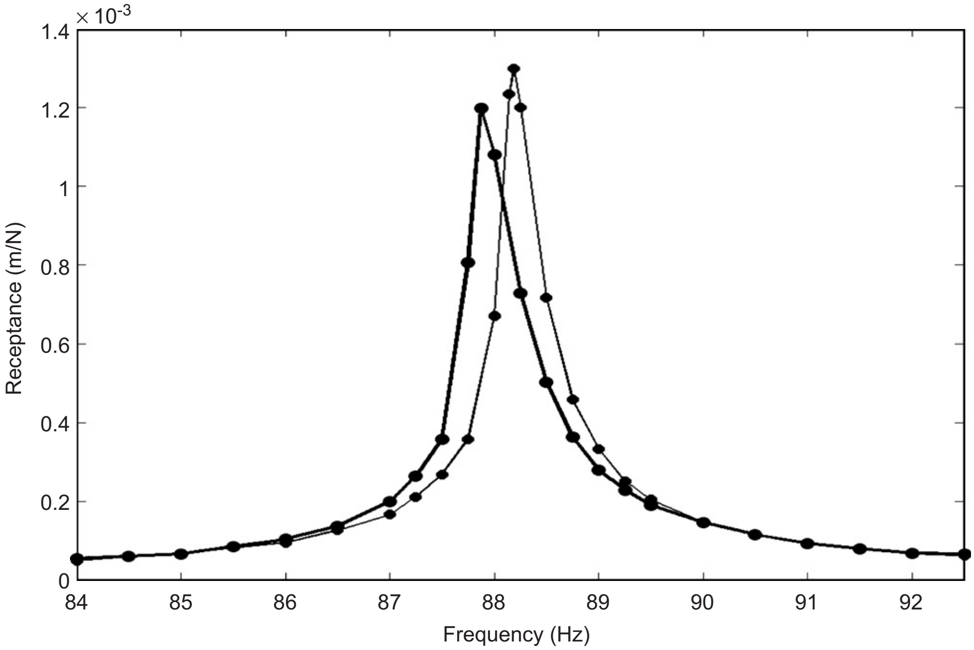

Fig. 8. Receptance around the first resonance: linear (solid line) and non-linear (tick solid line).

sensitivity coefficients with and without tip mass, one may realise the significance of the tip mass in introducing the effects of k_w in modes one and three and the effects of k_θ in mode two.

4.2. Identification of joint non-linear effects

In the second stage of experiments the structure is excited using a sinusoidal force at frequencies around the first and second resonances to record the non-linear frequency response curves of the structure. The sinusoidal

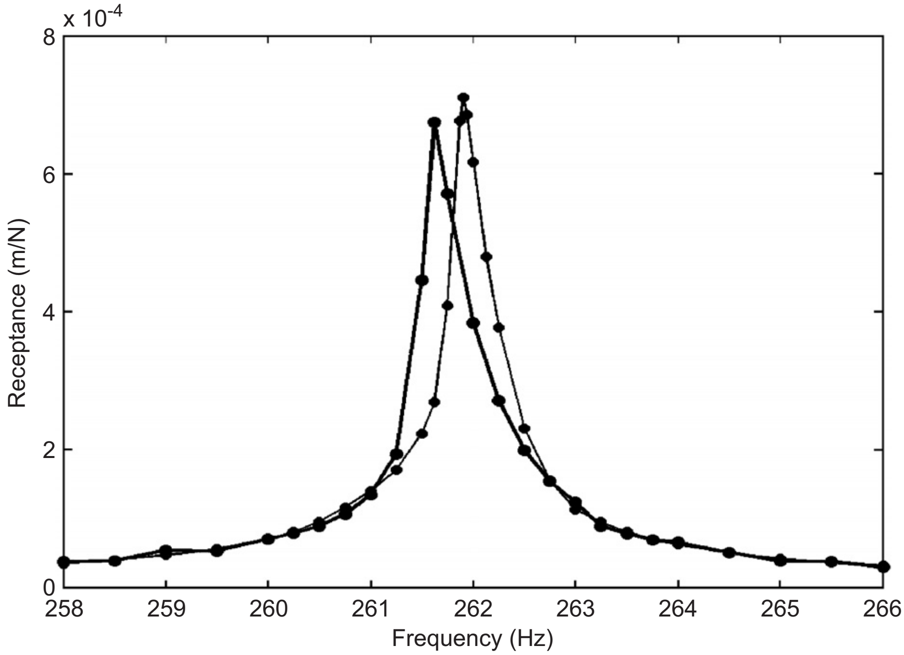

Fig. 9. Receptance around the second resonance: linear (solid line) and non-linear (tick solid line).

excitations are performed around each resonance point within frequency range of 10 Hz and with resolution of 0.5 Hz. Extra measurement points are chosen close to the resonance frequencies to be able to demonstrate the softening effects more clearly. The excitation forces in these tests are selected at two different levels to produce the linear and non-linear frequency response curves. The linear response curves are measured using a sinusoidal excitation with the amplitude of 44 mN while the amplitude of excitation force for measuring the non-linear response curve is increased to 130 mN. The receptance curves are shown in Figs. 8 and 9 and will be used to identify the generic parameters of the proposed joint model.

The non-linear stiffness parameters, k_{wN} and $k_{\theta N}$, and the damping coefficients, c_w and c_θ, are identified by minimising the following objective function:

$$\min \sum_{r=1}^{2} \left(W_{1r} \log\left|\alpha_{xr}(\omega) - \alpha_{0r}(\omega)\right| + W_{2r}(\omega_{xr}^2 - \omega_{0r}^2) \right) \tag{25}$$

where vectors $\alpha_{xr}(\omega)$ and $\alpha_{0r}(\omega)$ are the measured and the predicted non-linear receptance data around the rth mode, ω_{xr}^2 and ω_{0r}^2 are the rth measured and predicted resonances, and W_{1r}, W_{2r} are the weighting coefficients. The weighting functions are selected based on the confidence level on the accuracy of measured data points. In a large frequency response data set higher weighting functions are assigned to the data close to the resonance frequencies to capture the non-linear effects. In this study, only a small group of data points near resonance frequencies are used in identification procedure and the weighting functions are set to one indicating the same level of confidence in all theses data points.

The identification procedure starts with small assumed values for the unknown parameters followed by updating the non-linear joint parameters at each iteration step. At each iteration step the difference between the measured response and the predicted one is minimised. The model non-linear frequency responses are calculated using IHB method and only the prime harmonics are considered in the solution as no other harmonics were observed in the measurements. The objective function is minimised using the unconstraint minimisation strategy of unidirectional search method. The initial values of the four unknown parameters are kept small such that the response in the initial iteration stages is close to the linear response of the system. The optimisation procedure converges after 15 iterations and results the identified parameters tabulated in Table 3.

Table 3
The identified joint parameters

K_w (N/m)	K_θ (N/rad)	K_{wN} (N/m^3)	$K_{\theta N}$ (N/rad^3)	C_w (Ns/m)	C_θ (Ns/rad)
1.993×10^6	1.568×10^3	1.412×10^{14}	1.995×10^7	0.5	0.055

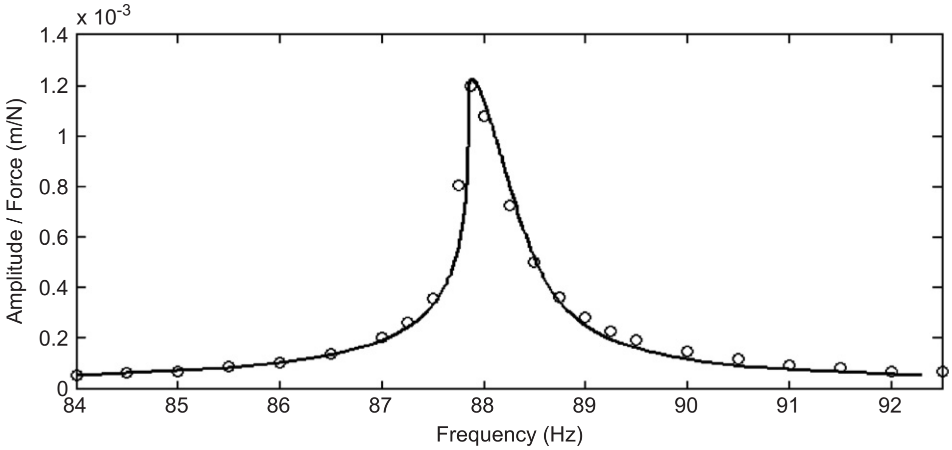

Fig. 10. Frequency responses around the first resonance, predicted (solid line) and measured (circles).

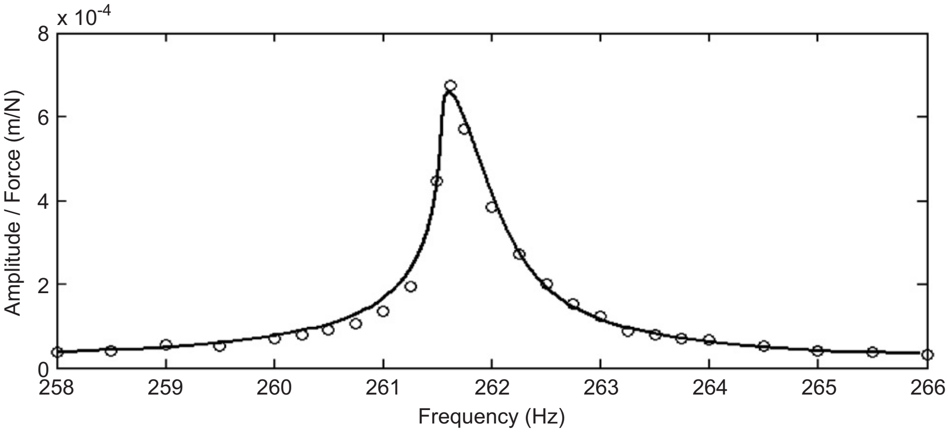

Fig. 11. Frequency responses around the second resonances, predicted (solid line), measured (circles).

Figs. 10 and 11 show the model frequency response predictions in vicinity of the first and second resonances, obtained using the identified parameters. The experimental measured responses are also demonstrated in these figures. The two sets of predicted and observed responses demonstrate good agreements indicating the model successfully defines the physics involved in the bolted lap joint.

Another measure of model evaluation is comparison of hysteresis curves at resonance frequencies; it is in these frequencies that the damping mechanism shows its strongest effects on the dynamics of the system. The predicted and measured hysteresis loops at first and second resonant frequencies of non-linear system are shown in Figs. 12 and 13. Again there are good agreements between the predicted and the observed results.

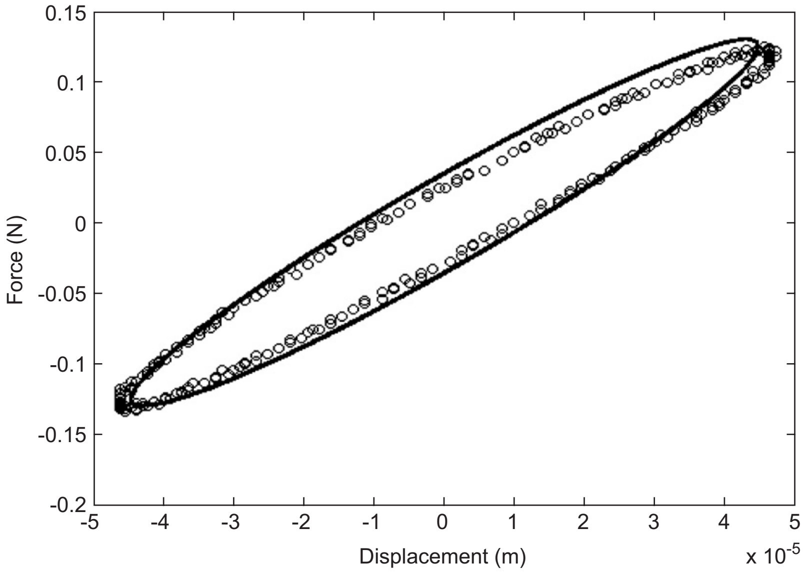

Fig. 12. Hysteresis loop at first resonance frequency (87.75 Hz), predicted (line) test (circle).

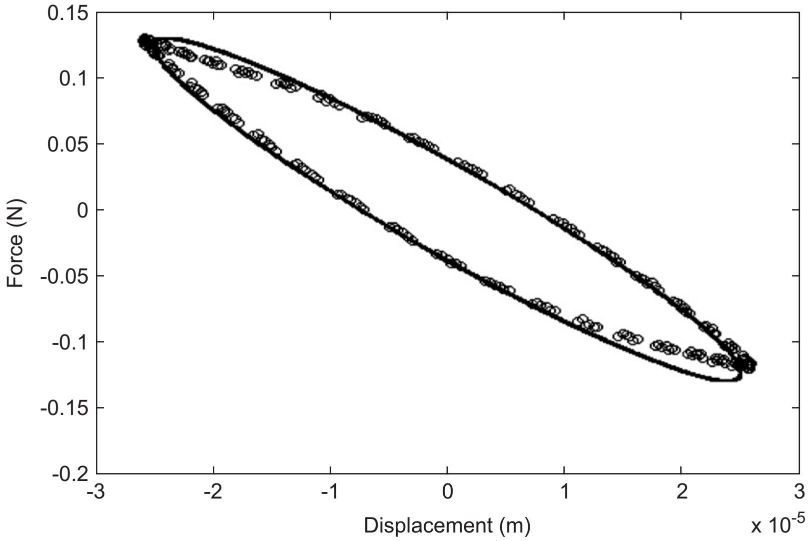

Fig. 13. Hysteresis loop at second resonance frequency (261.75 Hz), predicted (line) test (circle).

5. Conclusion

A generic element formulation is presented to model the non-linear behaviour of a single lap bolted joints. The generic element model consists of a generic stiffness and a generic damping matrix which are capable of representing the non-linear constitutional relations in a joint interface. The mass matrix of the joint interface model is assumed to be known and the linear and non-linear parameters of the generic stiffness and damping matrices are identified by minimising the difference between the model predictions and the measured responses. An approximate non-linear solution for the model is obtained using the method of IHB. Some tests are performed on a beam-like structure with a single bolted lap joint in the middle to compare the model responses with the experimental observations. The generic element parameters are identified by minimising the difference between responses of the experimental model and the analytical one. Good agreement between the model predictions and the observed behaviour of the system is observed indicating the generic element is capable of modelling the physics involved in the joint interface accurately. The concept of generic element for modelling joint interfaces can be easily extended to other type of joints to provide simple and efficient models for prediction of dynamical behaviour in assembled structures.

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