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Identification of nonlinear bolted lap joint models

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ABSTRACT

Lap joints have significant influence on the response of structures due to their localised nonlinear stiffness and damping. In this paper, dynamic behaviour of bolted joints is modelled and their parameters are identified using experimentally measured data. A thin layer of virtual elasto-plastic material is used to model the joint contact interface. In identification of the thin layer parameters, nonlinear responses measured at constant force amplitudes are used. The identified model predictions at different load levels are compared with corresponding measured responses. Good agreement is achieved between the two sets indicating successful parameterisation and identification of the joint interface model.

1. Introduction

Today finite element analysis plays an important role in engineering problems. Most of structures are assemblies of parts connected together with joints. Bolted lap joints are commonly used in structures and introduce some uncertainty to the structure model. To obtain an accurate finite element model of the structure, the effect of bolted joints should be modelled properly. Many research works have been focused on this subject. Mackerle [1] presented a review for finite element analysis of various types of joints under different loading conditions published between 1990 and 2002. Some works are focused on static behaviour of the bolted joints [2] while others have considered effects of bolted joints in dynamic response of the structure. In dynamic behaviour, the bolted joint nonlinear friction phenomena affect the response. Friction causes energy dissipation in the structure and consequently adds more damping to the dynamic response. Crocombe et al. [3] and Wang et al. [4] have investigated energy dissipations in bolted joints using finite element analysis. De Benedetti et al. [5] have represented friction as a nonlinear force in FE model to estimate damping due to bolted or riveted joints. Small loading cause infinitesimal deformation in the structure thus nonlinear effects of the joints can be linearised.

Ahmadian et al. [6] have updated parameters of surface-to-surface contact region in a finite element model within the linear behaviour range. Also Yang et al. [7] and Cunha et al. [8] have identified joint parameters using linear experimental data. By increasing the amplitude of external load, the behaviour of joint turns nonlinear. Many researchers have investigated this phenomenon by means of analytical methods and FE modelling. Ibrahim and Pettit [9] have presented a review on dynamic problems of bolted joints and other fasteners. Gaul and Lenz [10] have implemented Valanis model as a nonlinear substructure module into the finite element models. Some researchers have used contact elements to model nonlinear behaviour of joints in finite element analysis. Generally using contact elements accompanies with huge computations. Mayer and Gaul [11] have used segment-to-segment contact elements to define the contact stiffness of fixed joints for FE model updating.

It is observed the joint damping is independent of excitation frequency and is strongly dependent of vibration amplitude [12]. Accordingly, the principle of constant hysteresis can be used for the joint interface modelling [13]. In this paper, nonlinear behaviour of bolted joints is modelled using a thin layer of elasto-plastic material which obeys the constant hysteresis principle and its parameters are identified by nonlinear FE model updating. Since in fixed joints the interfaces are always in contact and restricted to small relative movements, contact search algorithms can be avoided and consequently save much computing time. The method provides a practical approach in modelling bolted joints experiencing a wide range of force excitations both in linear and nonlinear zones.

The thin layer elastic material properties represent the joint linear behaviour at low vibration levels and the plastic behaviour parameters model softening effects and frictional damping of the bolted joint. A stress–strain relation for the contact zone is selected that can represent the elastic and the plastic behaviour. Additionally this model introduces a parameter that can control the maximum curvature of stress–strain curve. Tuning the latter parameter properly, the transition from sticktion to micro-slip and macro-slip is modelled accurately.

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To identify the contact model parameters, a test setup was prepared and experimental frequency responses of the structure in both of linear and nonlinear regimes were measured. Finally model updating is performed using a direct search optimisation algorithm and in two separate stages parameters representing linear and nonlinear joint behaviour are identified. The identified model is validated against different sets of experimentally measured responses and good agreement between measurement and model responses are found.

The remainder of the paper is organised as followings. Section 2 discusses the lap joint modelling using zero thickness and thin layer elements. Guidelines on selecting parameters of thin layer elements are also provided. In Section 3, an experimental case study is presented consisting of a clamped beam assembly with a bolted lap joint in the middle. The beam is excited using a concentrated harmonic force and its response and excitation force signals are measured. Section 4 considers employing experimental results in FE model and obtaining the thin layer linear material properties. In Section 5, the material parameters representing nonlinear behaviour of joint interface model are identified using experimental frequency responses.

2. Joint contact interface

Distributed joint models are commonly employed in assembled structures when size of the contact joint is considerable in comparison to the smallest structural vibration wave length. There are two approaches used to model bolted joint contact interface as distributed joint models; contact elements and thin layer of special material. The contact elements or so called zero-thickness contact elements show joint sliding by an incremental quasi-linear constitutive equation that each increment can be defined as [6]:

$$\begin{Bmatrix} \Delta\sigma \\ \Delta\tau_1 \\ \Delta\tau_2 \end{Bmatrix} = \begin{bmatrix} k_n(1-\mu^2 k_n^2/H) & -k_n k_s \beta_1 \mu/H & -k_n k_s \beta_2 \mu/H \\ & k_s(1-\beta_1^2 k_s/H) & -k_s^2 \beta_1 \beta_2/H \\ Symm & & k_s(1-\beta_2^2 k_s/H) \end{bmatrix} \begin{Bmatrix} \Delta v \\ \Delta u_1 \\ \Delta u_2 \end{Bmatrix} \quad (1)$$

where $\Delta\sigma$, $\Delta\tau_i$ are, respectively, the incremental normal and tangential stresses, Δv and Δu_i are the incremental relative normal and tangential displacements across the joint. The parameters k_n and k_s are coefficients which penalise surface penetration and slipping, respectively. In Eq. (1) parameters H, β_i and μ can be defined as:

$$H = k_n\mu^2 + k_s, \quad \beta_i = \frac{\tau}{\sqrt{\tau_1 + \tau_2}}, \quad \mu = \tan\varphi \quad (2)$$

and φ is the friction angle. The coefficients of distributed contact stiffness, i.e. k_n and k_s, are dependent on the joint configuration and should be estimated. The nonlinear Eq. (1) also suggests an elasto-plastic constitutive equation. However the use of contact elements in dynamic analysis increases computation cost.

A thin layer of elasto-plastic material is another approach to model the contact layers such as bolted joints. Use of this thin layer of elasto-plastic material, in comparison to contact elements, leads to less computational costs. The joint interface between two contacting bodies can be modelled with continuous elements of very small but finite thickness. Thin layer elements were originally presented to model interfaces between rocks [14]. On aspect of employing the thin layer element is their aspect ratios. Depending on the element formulation on may employ elements with large aspect ratios without significant numerical errors [15]. Sensitivity of the model response to the aspect ratio of thin layer element needs to be checked by the analyst as it varies dependent on the element model.

A stress–strain relationship for the contact zone can be implemented into thin layer element model. The constitutive model is written generally in the form:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{12} \\ \tau_{23} \\ \tau_{31} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ & & c_{33} & c_{34} & c_{35} & c_{36} \\ & & & c_{44} & c_{45} & c_{46} \\ & & & & c_{55} & c_{56} \\ Sym. & & & & & c_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{31} \end{Bmatrix} \quad (3)$$

The number of unknown parameters of c_{ij} can be reduced by proper physical assumptions. Mayer and Gual [11] proposed the following assumptions regarding these parameters:

1. The off-diagonal terms are zero as there is no transversal contraction invoked by the contact interface.
2. Since the interface has no stiffness parallel to the joint's surface, the terms c_{11} and c_{22} disappear.
3. c_{33} represents the normal stiffness, whereas $c_{55} = c_{66} = G$ define the tangential stiffness of the joint.
4. Since the joint exhibits no stiffness for in-plane shearing, c_{44} is also zero.

A bolted joint under tangential loading experiences three distinct load–displacement regimes of sticktion, micro-slip and macro-slip. While the joint behaviour in stick state is linear micro-slip and macro-slip are nonlinear phenomena. To model the joint interface micro-slip and macro-slip behaviour, thin layer material is assumed to have elasto-plastic behaviour. The nonlinear behaviour of bolted joint is represented by nonlinear stress–strain curve of contact zone thin layer. The constitutive equation of thin layer is adopted in this study as:

$$\sigma = \left(E_p + (E_e - E_p)\left(1 + \left(\frac{|(E_e - E_p)\varepsilon|}{S_y}\right)^n\right)^{-\left(\frac{1}{n}\right)} \right)\varepsilon \quad (4)$$

The joint contact model has four unknown parameters: elastic modulus E_e, plastic section modulus E_p, yield stress S_y and parameter n that determines smoothness of transition curvature from micro-slip to macro-slip behaviour. Eq. (4) was originally defined in [13] to analyse vibration and deflection characteristics of semi-rigid jointed frames.

Eq. (4) is employed in this study to define c_{55} and c_{66} with four parameters. These parameters can be obtained using analytical methods and are known to be functions of contact surface preload, surface roughness and the contacting body materials.

3. Experimental case study

The application of proposed joint interface modelling using thin layer elements is demonstrated using an experimental bolted lap joint. The experimental set up consists of two identical steel beams coupled via a bolted lap joint as shown in Fig. 1. Each beam had a length of 235 mm, width of 25.4 mm and thickness of 6 mm. The assembled structure geometrical parameters were L = 420 mm (overall length), and S = 50 mm (lap joint length). The structure had fixed-free boundary conditions and it was excited by a concentrated external force applied by an electromagnetic shaker through a stinger. The force applied to the structure was measured with force transducer. The details of controlled force excitation to extract linear and nonlinear joint interface behaviour are given in a previous work [12].

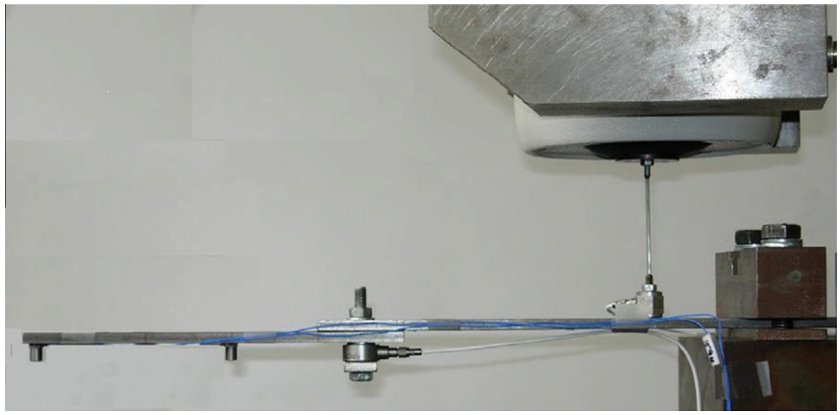

Fig. 1. The test setup [12].

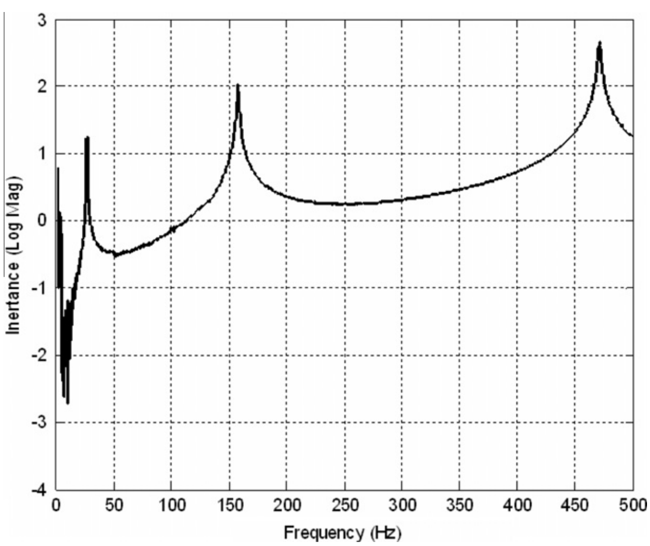

Fig. 2. Linear measured FRF [12].

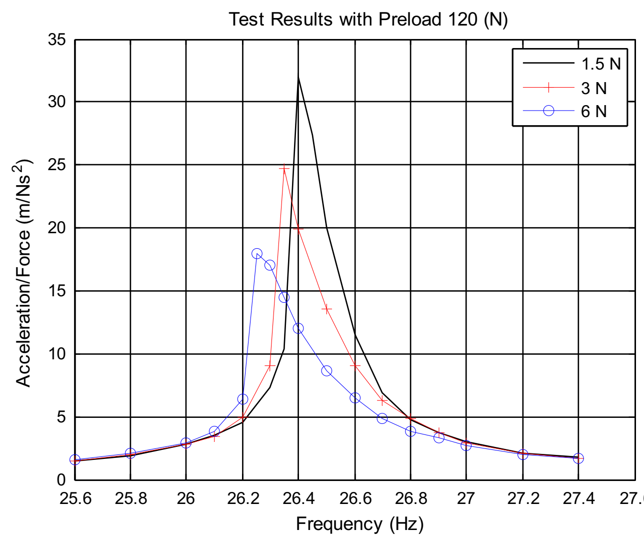

Fig. 3. Nonlinear measured FRFs at different force levels [12].

Table 1
First three measured natural frequencies (Hz).

ω_1	ω_2	ω_3
26.4	157.6	471.8

First the structure was excited by using a low-level random excitation signal and linear frequency response functions (FRFs) were measured. A resultant linear FRF is shown in Fig. 2 and Table 1 shows the corresponding three first experimental natural frequencies. The linear natural frequencies are used to update a linear finite element model of the beams and linear material properties in thin layer of bolted joint.

In the second stage of experiment, the structure was excited using a single harmonic force, the excitation frequency being chosen to vary in a frequency band close to the first natural frequency. The excitation amplitude was maintained at a constant level for all excitation frequencies, the steady state response of the structure and its corresponding force signal were recorded and the nonlinear FRFs obtained. A force cell placed under the bolt head made it possible to measure the bolt preload and to carry out the experiment for different known preloads. The experiments were completed for bolt preload 120 N. At this preload three different excitation levels were used, $F = 1.5$ N, $F = 3$ N and $F = 6$ N. These are shown in Fig. 3 and correspond to the accelerometer closest to the tip of the beam. The measured preload was found to remain unchanged after each experiment, which seems reasonable since the preload was high, the frequency was low and the applied sinusoidal forces were low. The nonlinear joint interface parameters were identified from these test results.

4. Linear finite element model

A FE model of the assembled structure is developed using MSC/NASTRAN software to predict its dynamical behaviour. The FE model of structure is shown in Fig. 4. The beam substructures are modelled using two node beam elements and the bolted joint has been modelled by a thin layer of elasto-plastic material. The 2D layer is modelled using QUAD4 shell elements. The rectangular shell elements have heights and widths equal to the length of beam elements and the beam thickness, respectively. The interface layer thickness is chosen to fill the distance between the neutral axes of the beams at the interface region. The beams have equal thicknesses and the distance is equal to the thickness of beams. This choice provides not only the correct flexural bending stiffness in the joint region but also provides appropriate mass distribution for the model. The mass effects of the bolt and nut are also modelled using lumped nodal masses.

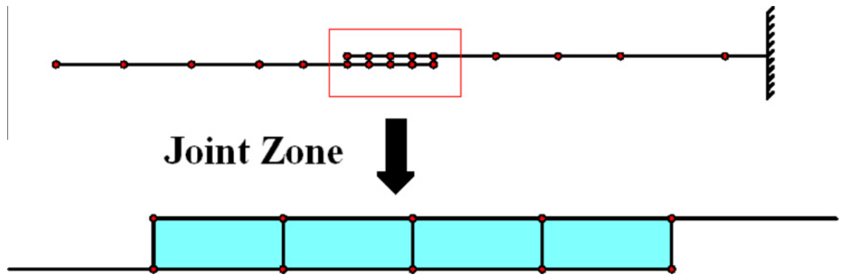

Fig. 4. FE model of the test structure.

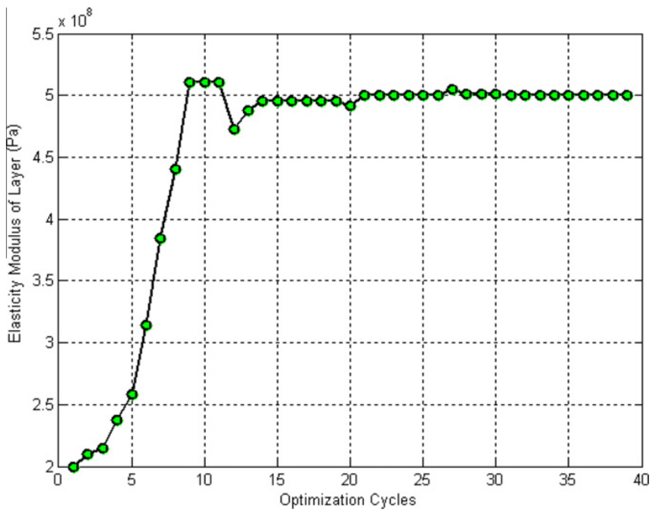

Fig. 5. Variation of parameter E_e in optimisation process.

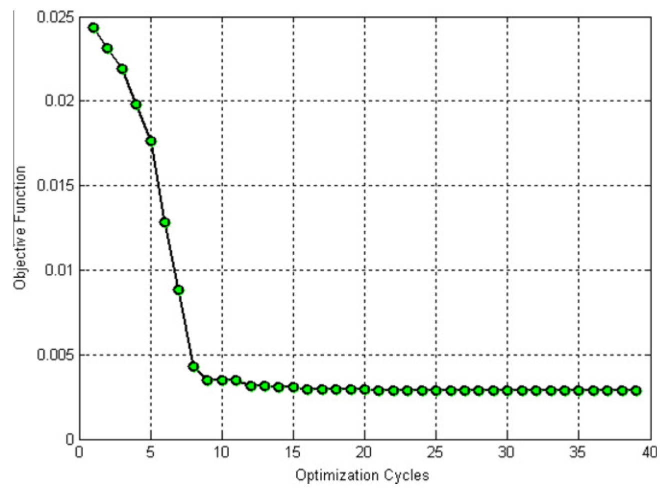

Fig. 7. Variation of objective function in optimisation process.

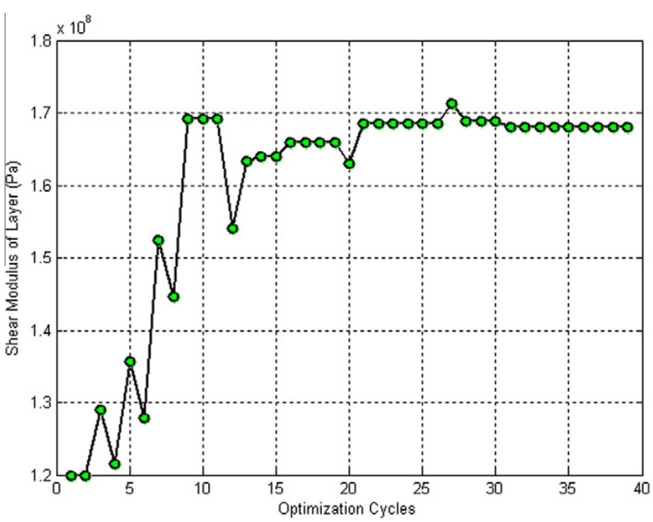

Fig. 6. Variation of parameter G in optimisation process.

Table 2
Updated elastic parameters.

Parameter	E_e (MPa)	G (MPa)
Value	499.82	168.13

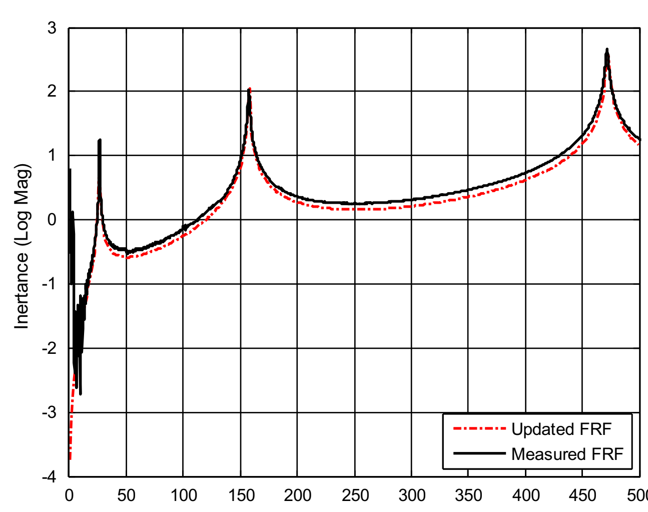

Fig. 8. Measured and updated linear FRFs.

It was assumed the model of the beam sections was correct and that the structure formed an ideal cantilever with fixed-free end conditions. The deviations between FE model predictions and the experimental observations were assumed to be associated to the thin layer element material properties, i.e. modulus of elasticity and its shear modulus. Avoiding the excess mass effect, the density of the material adopted the joint layer is set to a small value (1 kg/m^3).

It is assumed the thin layer material in linear state is isotropic and can be characterised using any pair of the three material parameters namely, modulus of elasticity, Poisson's ratio and its shear modulus. These parameters are identified by minimising

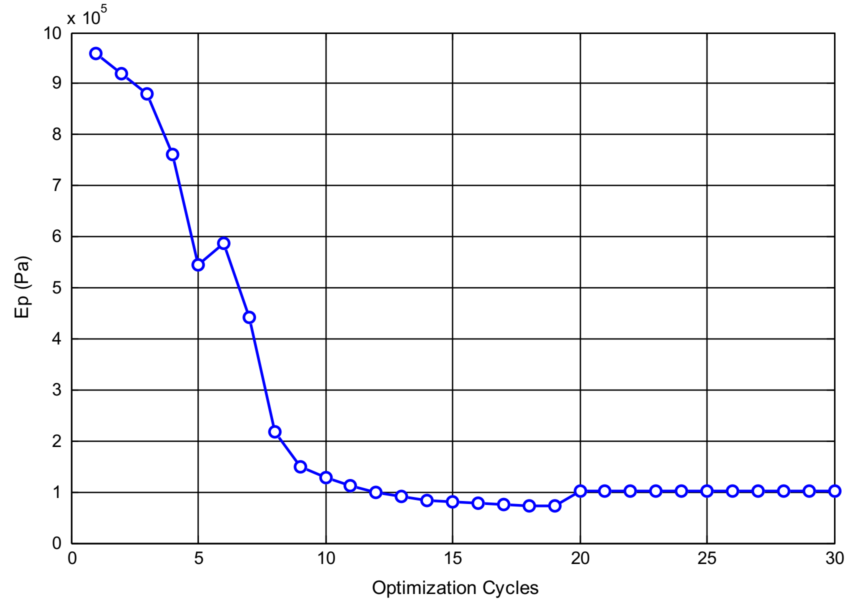

Fig. 9. Variation of parameter E_p in optimisation process.

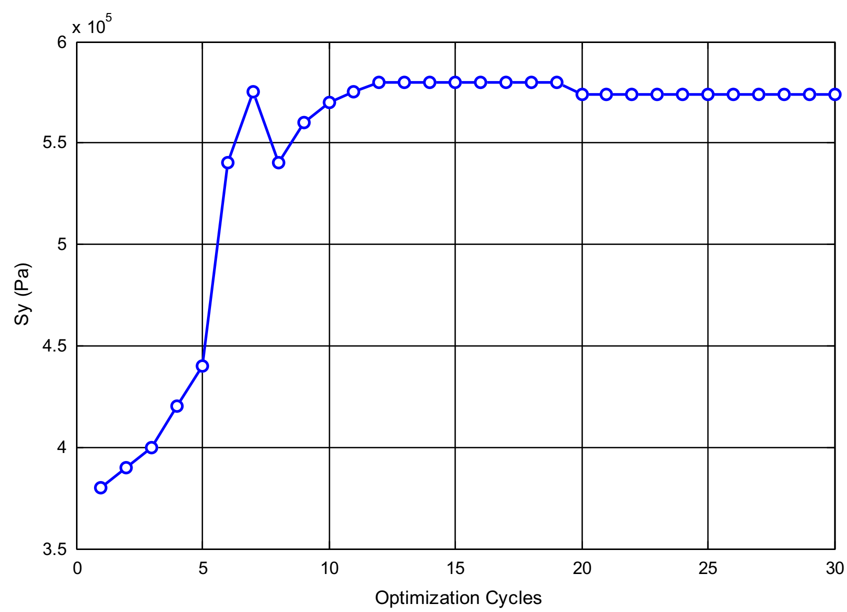

Fig. 10. Variation of parameter S_y in optimisation process.

the difference between measured and FE model predicted natural frequencies, i.e.:

$$\min\left(\sum_{i=1}^{3}\left|\frac{\omega_{0i}-\omega_{ei}}{\omega_{ei}}\right|\right) \quad (5)$$

where ω_{ei} and ω_{0i} are measured and predicted natural frequencies. The design parameters were selected as modulus of elasticity and shear modulus as they are non-zero, real positive values. Poisson's ratio may change sign or becomes zero during optimisation procedure and causes numerical instabilities in the procedure and hence was not selected as a design parameter (details are provided in Appendix A).

Minimisation of objective function (5) is performed using iterative linear Eigen-sensitivity method. The choice of initial parameters is made based on previous observations [6] indicating the interface equivalent material properties are 3–4 orders of magnitude less than their corresponding values of contacting bodies. The variations of objective function and design parameters during the optimisation process are shown in Figs. 5–7. These figures indicate smooth convergence of the optimisation algorithm and Table 2 shows the identified parameters in the final optimisation stage.

The measured and linear updated model FRFs are shown in Fig. 8. The updating process provided good agreement between FE model predictions and the test results.

Next the nonlinear behaviour of the test structure is used to identify the nonlinear joint interface parameters.

5. Joint model identification

In order to predict the nonlinear behaviour of the test setup, the Nonlinear Transient module of NASTRAN software is employed to

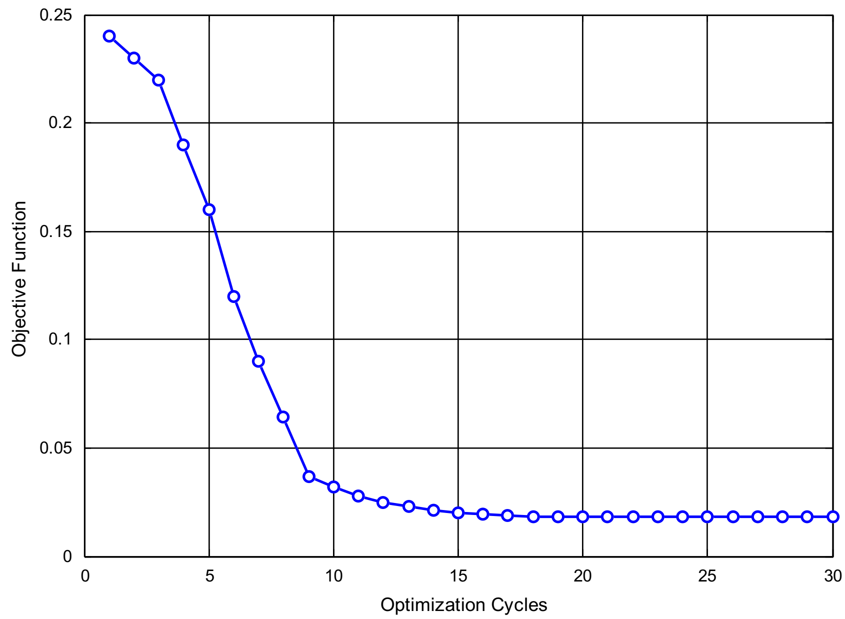

Fig. 11. Variation of objective function in optimisation process.

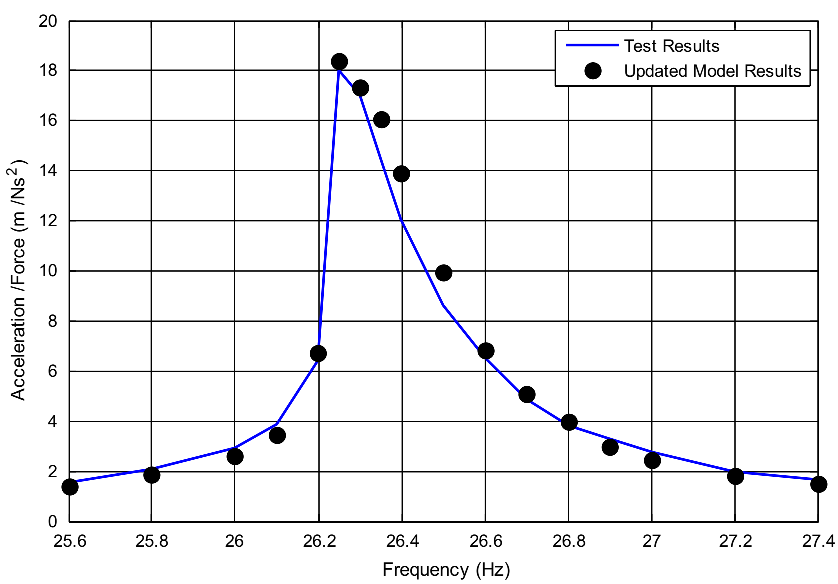

Fig. 12. Measured and updated model FRFs at 6 N.

Table 3
Identified nonlinear parameters.

Parameter	E_p (Pa)	S_y (Pa)	n
Value	1.010e5	5.7426e5	2.5

run the model. In this practice, the model was excited with harmonic forces corresponding to those in the experiment and nonlinear FRFs of the model are obtained. These FRFs are generated based on initial values set for the nonlinear thin layer material model.

The material behaviour introduced in Eq. (1) has four unknown parameters. One of them is modulus of elasticity E_e which is identified in previous section along with elastic shear modulus, G, to represent linear response of structure. In the next stage, the remaining three parameters which represent the nonlinear behaviour of the joint, namely macro-slip limit, S_y, the stiffness at macro-slip, E_p, and the transition from stick to micro-slip and macro-slip n were determined.

The parameter n controls smoothness of transition from stick to micro-slip and macro-slip behaviour. This variable has little influence on the FRF but significantly influences the hysteresis loop derived from excitation harmonic force and the response of test setup. Therefore this parameter is obtained from these hysteresis loops.

The other two parameters E_p and S_y were determined by minimising the difference between the measured and predicted nonlinear FRFs. To define the objective function only few points in the vicinity of FRF picks were selected as these are the response harmonics sensitive to nonlinear behaviour of joint. The objective function defined as:

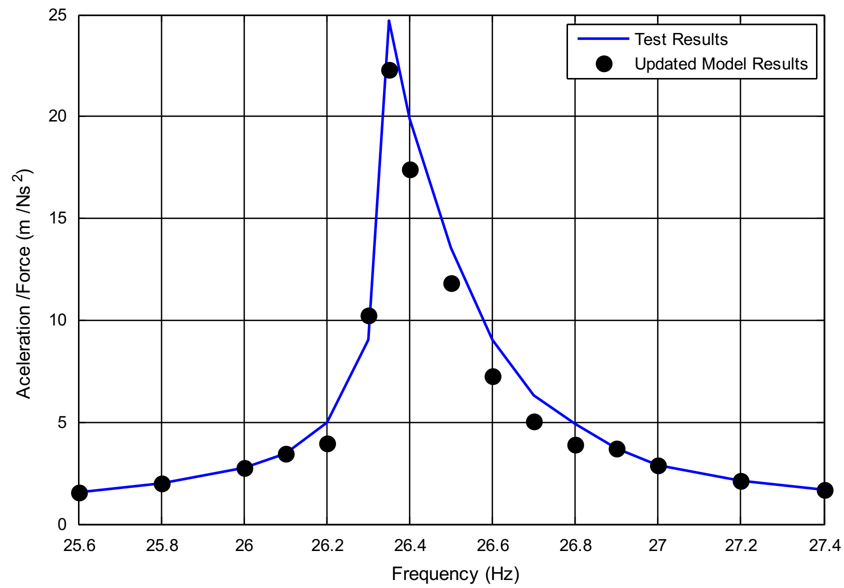

Fig. 13. Measured and updated model FRFs at 3 N.

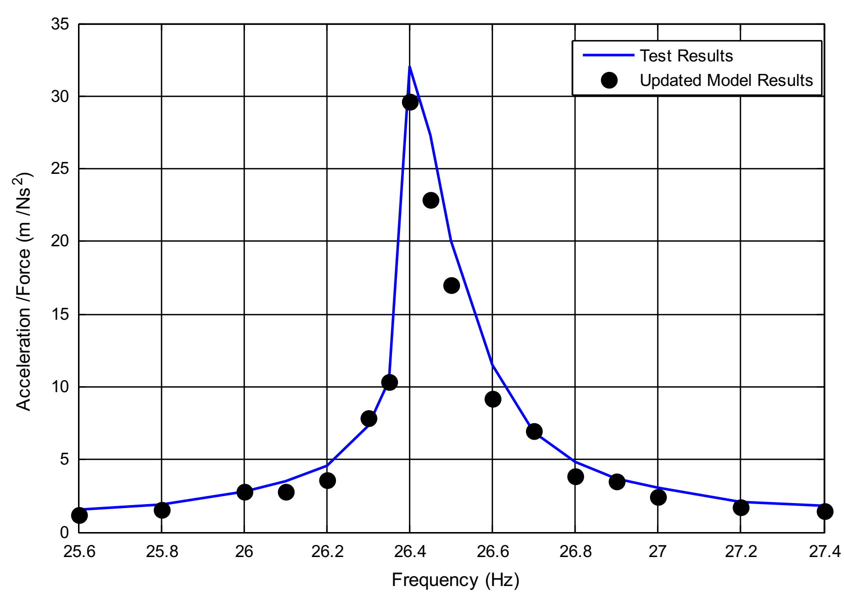

Fig. 14. Measured and updated model FRFs at 1.5 N.

$$\min \sum_{i=1}^{n} \frac{|\alpha_0(\omega_i) - \alpha_e(\omega_i)|}{\alpha_e(\omega_i)} \tag{6}$$

where α_0 and α_e are analytic and experimental FRFs at selected harmonics ω_i, $i = 1, 2, \ldots, n$. Figs. 9–11 show variations of the objective function and the design variables in optimisation process. The optimisation procedure was performed by importing the prediction of the model to MATLAB software. The objective function was formed in this software and its minimisation is performed using a direct search optimisation algorithm. The nonlinear model identification is performed using only the measured data of FRF obtained by harmonic force excitation of amplitude 6 N. Fig. 12 shows experimental measured FRF and updated model FRF. The remaining experimental data were used to validate the identified model. Identified values of nonlinear parameters are shown in Table 3.

The prediction of model with identified nonlinear model is obtained and the other two levels of force excitations of 1.5 N and 3 N. Figs. 13 and 14 compare the model response to nonlinear measured FRFs at these force levels. The results show good agreement between the model predictions and test observations indicating proper nonlinear modelling of the joint interface and successful identification of the contact interface parameters.

The joint interface model represents the flexural behaviour of contact region accurately in a wide frequency range. This enables the model to predict linear and nonlinear behaviour of the beam in bending regardless of the loading configuration. However in higher frequencies if loadings cause axial or torsional movements in the contact interface there would be some uncertainties associated with the model predictions.

6. Conclusion

A thin layer interface model with virtual elasto-plastic material behaviour is adapted to model nonlinear effect of bolted joints in an assembled structure. The constitutive relation for the virtual interface material is capable of representing the joint interface behaviour in three different phases of sticktion, micro-slip and macro-slip. Parameters of the interface virtual material are unknown and are identified by minimising the difference between model predictions and experimental observations. The resultant model is validated by comparing its response predictions to observed experimental responses at different force levels. Good correlations between the experiments and model predictions are achieved. The proposed strategy in modelling nonlinear effects in mechanical joint interfaces is accurate, requires minimum computations and can be applied using existing FE programs.

Appendix A

Selecting the Poisson's ratio as a design variable causes numerical instabilities in the minimisation procedure. The source of instability in optimisation problem is reversing the gradient of sensitivity matrix when Poisson's ratio goes to zero. This can be seen from the followings.

The stiffness matrix of interface element is derived as:

$$K = \iint_A B^T D B \, dA$$

where A is the element area, matrix B relates the displacement vector to the element strains and matrix D defines the constitutive law of the element. In plane stress state D takes the following form:

$$D = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ & 1 & 0 \\ Sym. & & 1-\nu \end{bmatrix}$$

Sensitivity of stiffness matrix is directly related to the sensitivity of matrix D to the material properties:

$$\frac{\partial D}{\partial E} = \frac{1}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ & 1 & 0 \\ Sym. & & 1-\nu \end{bmatrix},$$

$$\frac{\partial D}{\partial \nu} = \frac{E}{(1-\nu^2)^2}\begin{bmatrix} 2\nu & 1+\nu^2 & 0 \\ & 2\nu & 0 \\ Sym. & & 2\nu-\nu^2-1 \end{bmatrix}$$

It is seen the sign of first two diagonal terms in $\partial D/\partial \nu$ change as the sign of ν is changed. In the minimisation procedure this causes oscillations of the solution around $\nu = 0$ and does not allow a global minimum solution.

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