Mechanical Systems and Signal Processing 23 (2009) 2008–2018

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/jnlabr/ymssp

Identification of nonlinear boundary effects using nonlinear normal modes

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ARTICLE INFO

Article history:
Received 29 February 2008
Received in revised form
10 May 2008
Accepted 30 July 2008
Available online 5 August 2008

Keywords:
Boundary condition
Local nonlinearity
Nonlinear normal mode
Eigensensitivity

ABSTRACT

Local nonlinear effects due to micro-slip/slap introduced in boundaries of structures have dominant influence on their lower modal model. This paper studies these effects by experimentally observing the behavior of a clamped–free beam structure with local nonlinearities due to micro-slip at the clamped end. The structure is excited near one of its resonance frequencies and recorded responses are employed to identify the nonlinear effects at the boundary. The nonlinear response of structure is defined using an amplitude-dependent nonlinear normal mode identified from measured responses. A new method for reconstructing nonlinear normal mode is represented in this paper by relating the nonlinear normal mode to the clamped end displacement-dependent stiffness parameters using an eigensensitivity analysis. Solution of obtained equations results equivalent stiffness models at different vibration amplitudes and the corresponding nonlinear normal mode is identified. The approach results nonlinear modes with efficient capabilities in predicting dynamical behavior of the structure at different loading conditions. To evaluate the efficiency of the identified model, the structure is excited at higher excitation load levels than those employed in identification procedures and the observed responses are compared with the predictions of the model at the corresponding input force levels. The predictions are in good agreement with the observed behavior indicating success of identification procedure in capturing the physical merits involve in the boundary local nonlinearities.

1. Introduction

In order to identify the characteristics of boundaries, experimental identification techniques are employed, and numerous methods have been proposed [1–3]. In these proposals the structure and its associated boundary conditions are assumed to behave linearly and well-established linear identification techniques are employed to produce a model capable of predicting the boundary effects within limited excitation force levels. However, in practical cases with higher excitation levels the boundary conditions often become nonlinear due to clearance, friction, and so on. In higher vibration levels micro-slips are common phenomenon at the boundary interfaces and dominate the structure behavior at low frequencies. The common effects of micro-slips in the structure behavior are the stiffness softening, which manifest itself in the decrease of the resonance frequency as the amplitude of excitation force increases, jump phenomenon in the frequency response curve and dissipation of energy, which significantly reduces the vibration amplitude at resonance frequency.

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0888-3270/$ - see front matter
doi:10.1016/j.ymssp.2008.07.013

Nomenclature

Subscripts

i, j, r, s	indexes representing elements of a matrix or a vector

Superscripts

$\sim$	implies the nonlinear nature of a variable, e.g. $\tilde{\omega}$

Roman letters

E	error due to approximating the nonlinear system with a linear equivalents system
N	total number of degrees of freedom (DOFs) in a system
T	period of oscillation
h	imaginary part of stiffness parameter
k	real part of stiffness parameter
q_r	contribution of mode r in the response
u_i	displacement of DOF i

Greek letters

Ω	excitation frequency
β_j	coefficients of eigenvalue variation function
γ_{ij}	coefficients of nonlinear normal mode variation functions
η	damping factor
ω	eigen frequency

Matrices and vectors

$\lfloor J_\omega \rfloor$	row vector sensitivity of eigenvalue with respect to stiffness parameters
$\lfloor J_\phi \rfloor$	sensitivity matrix of nonlinear normal mode with respect to stiffness parameters
$[K]$	stiffness matrix
$[M]$	mass matrix
$\{P\}$	excitation vector
$\{U(\Omega)\}$	vector of frequency response function
$\{f(u)\}$	vector of nonlinear restoring forces
$\{u\}$	vector of system's response
$\{\phi_r\}$	vector of normal mode r.

In contrast with the well-known methods for linear structures, the methods available for analysis of nonlinear systems are generally more complicated, restricted to specific types of nonlinearity, and applicable to systems with limited degrees of freedom (DOFs). Traditional techniques for analyzing the dynamics of nonlinear structures are based on the assumptions of weak nonlinearities and of a 'nonlinear' modal structure that is similar or a small perturbation of the underlying linearized system.

The nonlinear normal modes provide a rigorous theoretical framework for extending modal analysis to nonlinear systems. It is therefore attractive to develop nonlinear system identification techniques based on these nonlinear modes. The focus in the present paper is on the inverse problem, i.e. on the development of a nonlinear model in modal space from experimental measurements. To identify the boundary effects and representing them in modeling, one needs to employ appropriate techniques capable of capturing local nonlinear effects in the boundaries. The boundary conditions have dominant effects on the lower modes of a structure. Therefore identifying these nonlinear normal modes from experimentally observed data and employing them in predicting the structure response in the desired loading condition would be a natural choice.

The concept of normal modes for nonlinear systems was first introduced by Rosenberg [4,5] who described a geometrical method to obtain a qualitative study of nonlinear mode. Szemplinska-Stupnicka [6,7] showed that the mode of vibration in resonant conditions can be considered close to the nonlinear normal mode and used the Ritz procedure to determine the nonlinear natural frequencies and the nonlinear normal mode of structures with local nonlinearties as functions of modal amplitude. The numerical computation of these modal parameters involves a nonlinear eigenvalue problem generally solved using a Newton–Raphson procedure. By curve fitting forced responses in the neighborhood of a resonance, the nonlinear modal parameters can be obtained from experimental data. Based on this methodology, modal parameter identification procedures for forced response of nonlinear systems were developed by Jezequel [8], Setio et al. [9,10], and Chong and Imregun [11]. In these studies, it is assumed that the resonant frequencies are not close to each other, and the frequency response is expressed as a linear combination of contributions from resonant nonlinear normal modes. The coupled nature of the modal space is accounted for, by adding small contributions from the non-resonant modes. The modal parameters of the resonant nonlinear normal modes behave in a nonlinear manner with the modal amplitudes, and are identified by curve-fitting procedures. The non-resonant modal parameters are merely the linear modal parameters which are computed by standard modal analysis at sufficiently small amplitude motions. Identification of amplitude-dependent nonlinear modes using successive approximation model is proposed by Huang and Iwan [12]. Literature reviews on the properties and construction procedures of nonlinear normal modes can be found in [13,14].

Present study is the continuation of the works reported by Jezequel [8], Setio et al. [9,10], and Chong and Imregun [11]. The objective is the same, i.e. identifying the nonlinear normal modes from measured nonlinear responses but the procedure is different. In the present study, the variations in the nonlinear normal modes are defined as a function of local nonlinear parameters and not as a combination of the linear modes of the system as suggested by these authors [8–11].

The clamped end effects are modeled using a nonlinear complex-valued stiffness matrix. The parameters of the stiffness matrix are assumed to be displacement dependent to account for softening effects due to micro-slips in the boundary and are complex valued to include the displacement-dependent dissipation mechanism in the clamping joint. Identification of the nonlinear normal mode is performed by defining the mode as a function of the clamped joint parameters. The function is formed using eigensensitivity analysis of the nonlinear normal mode to the clamped joint parameters.

The paper is organized as followings. Identification procedure of nonlinear normal modes by invoking eigensensitivity is developed in Section 2. Then the performance of proposed nonlinear identification method is investigated in an experimental case study. The nonlinear responses are measured by exciting the beam using a fixed amplitude single harmonic force at individual frequencies near its second mode. The collected experimental data are used to identify variations of the nonlinear normal mode as a function of its contributions in the response amplitude. The identified nonlinear normal mode is then employed to predict the force responses of the structure at higher force levels than those employed in the identification procedure. The results presented in Section 3 show very good agreements between the model predictions and the experimental observations. Section 4 and 5 provides conclusions and references of the paper, respectively.

2. Identification of nonlinear boundary model

Behavior of a structure under harmonic excitations can be modeled using a set of nonlinear second order differential equations with N DOFs

$$[M]\{\ddot{u}\}+[K]\{u\}+\{f(u)\}=\{P\}\cos\Omega t \tag{1}$$

where vector $\{f(u)\}$ represents the nonlinear restoring forces at the clamped boundary. At very low-level excitation forces the nonlinear effects do not appear in the response and the structure behaves linearly. In these situations the normal co-ordinates are coupled only through structural damping terms. If the structural damping is negligible and the natural frequencies are not close to each other, the vector of frequency response functions can be approximated as

$$\{U(\Omega)\}=\sum_{r=1}^{N}\frac{\{\phi_r\}^T\{P\}\{\phi_r\}}{\omega_r^2-\Omega^2} \tag{2}$$

As the excitation force increases, micro-slips between clamping surfaces at the boundary accurse and the structure behave in a nonlinear manner. In nonlinear systems, however, the coupling of the normal co-ordinates is caused not only by damping (linear or nonlinear) but also by the terms representing the nonlinear parts of restoring forces. In this case the argument of "weak nonlinearity" is used to legitimize neglecting of the coupling of the normal co-ordinates [6,7]. Based on these assumptions a single nonlinear normal mode, $(\{\tilde{\phi}_s\},\tilde{\omega}_s(1+i\eta_s^2))$, is used to represent the nonlinear effects near resonant frequency of the system and contribution of modes far from excitation frequency, are small and these modes could be assumed to behave linearly, i.e.

$$\{U(\Omega)\}=\frac{\{\tilde{\phi}_s\}^T\{P\}\{\tilde{\phi}_s\}}{\tilde{\omega}_s^2(1+i\eta_s^2)-\Omega^2}+\sum_{\substack{r=1\\ r\neq s}}^{N}\frac{\{\phi_r\}^T\{P\}\{\phi_r\}}{\omega_r^2-\Omega^2} \tag{3a}$$

Consequently the time-domain response is defined as

$$\{u(t)\}=\{\tilde{\phi}_s\}q_s(t)+\sum_{\substack{r=1\\ r\neq s}}^{N}\{\phi_r\}q_r(t) \tag{3b}$$

The nonlinear normal mode $(\{\tilde{\phi}_s\},\tilde{\omega}_s(1+i\eta_s^2))$ is a function of the deformations in the boundary and the mode shape $\{\tilde{\phi}_s\}$ is uniquely defined as mass normalized. The following provides a method for identifying the relationships between the boundary deformations and the nonlinear normal mode $(\{\tilde{\phi}_s\},\tilde{\omega}_s(1+i\eta_s^2))$.

As depicted in Fig. 1, when one moves from point A to B on nonlinear frequency response curve, the response amplitude changes and nonlinear mechanisms of the structure in the boundary become active. This leads to changes in boundary clamping stiffness parameters and corresponding eigen properties of the system. One may assume an equivalent linear model for the system at point B as:

$$[M]\{\ddot{u}\}+[K+\tilde{K}(q_s)]\{u\}=\{P\}\cos\Omega t \tag{4}$$

where $\tilde{K}$ is a complex-valued matrix capable of creating equivalent stiffness and damping effects as the nonlinear part of the restoring force in the boundary. This matrix is a function of q_s, the contribution of the nonlinear normal mode in the response.

Based on Iwan's proposal [15] an equivalent linear model for $[\tilde{K}(q_s)]$ at each q_s level can be found by minimizing the residue

$$\{\varepsilon(q_s)\}=\{f(u)\}-[\tilde{K}(q_s)]\{u\} \tag{5}$$

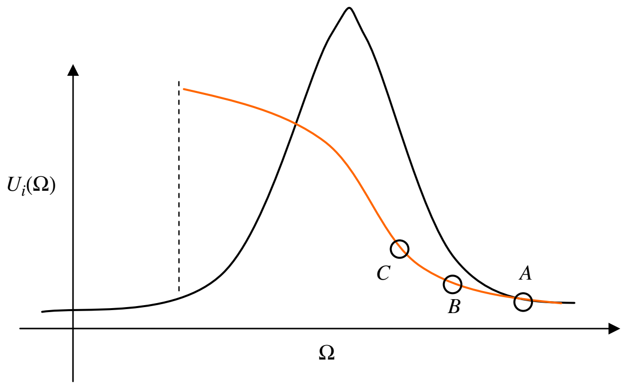

Fig. 1. Linear (black line) and nonlinear (red line) FRFs for a typical system.

The excitation force is harmonic therefore norm of residues over one cycle of motion is

$$E(\{\varepsilon\}) = \frac{1}{T}\int_0^T (\{\varepsilon\}^T\{\varepsilon\})\,dt \tag{6}$$

Necessary conditions for minimizing the norm of residues are

$$\frac{\partial}{\partial \tilde{k}_{ij}}E(\{\varepsilon\}) = 0, \frac{\partial}{\partial \tilde{h}_{ij}}E(\{\varepsilon\}) = 0 \tag{7}$$

If the nonlinear restoring force at each node could be thought of as representing a system of nodal points interconnected by nonlinear elements $\widehat{f}_{ij}(y_{ij})$, where $y_{ij} = u_i - u_j (i \neq j)$ and $y_{ii} = u_i$, whose behavior depends only upon the relative coordinates between these points, i.e.

$$f_i(u) = \sum_{j=1}^{N} \widehat{f}_{ij}(y_{ij}), \widehat{f}_{ij}(y_{ij}) = -\widehat{f}_{ij}(-y_{ij}) \tag{8a}$$

then the parameters $\tilde{K}_{ij}$ and $\tilde{h}_{ij}$, the real and imaginary parts of the equivalent linear stiffness, are obtained as

$$k'_{ij} = \frac{\int_0^T y_{ij}\widehat{f}_{ij}(y_{ij})\,dt}{\int_0^T y_{ij}^2\,dt}, h'_{ij} = \Omega\frac{\int_0^T \dot{y}_{ij}\widehat{f}_{ij}(\dot{y}_{ij})\,dt}{\int_0^T \dot{y}_{ij}^2\,dt}, \quad i,j = 1,\ldots,N \tag{8b}$$

In the study of boundary effects measuring the nonlinear part of the restoring force $\{f(u)\}$ and the deformations in the boundary is not practical. Therefore a new approach considering on these limitations are developed in this paper as followings. The proposal of this paper for identification of nonlinear boundary effects without the need for performing measurements in this part of the structure is based on expressing sensitivity of the measured nonlinear normal mode $(\tilde{\phi}_s, \tilde{\omega}_s)$ to the changes in the stiffness matrix parameters.

As shown in Fig. 1, the eigen properties at point B can be defined in terms of corresponding properties at point A using Taylor series expansion. Provided this step is sufficiently small, one may retain only the first order partial derivatives as:

$$\Delta\tilde{\omega}_s^2 = \sum_{i=1}^{m} \left.\frac{\partial\tilde{\omega}_s^2}{\partial k_i}\right|_A \Delta k_i = \lfloor J_\omega \rfloor\{\Delta k\} \tag{9a}$$

$$\Delta\tilde{\phi}_s = \sum_{i=1}^{m} \left.\frac{\partial\tilde{\phi}_s}{\partial k_i}\right|_A \Delta k_i = [J_\phi]\{\Delta k\} \tag{9b}$$

The variables k_i, $i = 1, 2, \ldots, m$ are the parameters of matrix $\tilde{K}$ and are assumed to be complex to take into account the displacement-dependent damping introduced by micro-slips at the boundary. At each response level the system is represented using a linearized stiffness model consisting of two parts: $[K]$ corresponding to the stiffness of linear parts of structure and $[\tilde{K}]$ the equivalent linearized stiffness representing the stiffness of those parts of structure behaving in a nonlinear manner. In order to determine the nonlinear normal modes $[K]$ is kept constant and only parameters of $[\tilde{K}]$ are allowed to change. Therefore, the eigensensitivity of the linearized model can be determined using the following analytical expressions

$$\frac{\partial\omega_s^2}{\partial k_i} = \frac{\{\phi_s\}^T(\partial[\tilde{K}]/\partial k_i)\{\phi_s\}}{\{\phi_s\}^T[M]\{\phi_s\}} \tag{10a}$$

$$\frac{\partial\{\phi_s\}}{\partial k_i} = \sum_{\substack{r=1\\ r\neq s}}^{N} \frac{\{\phi_r\}^T(\partial[\tilde{K}]/\partial k_i)\{\phi_s\}}{(\omega_s^2-\omega_r^2)}\{\phi_r\} \tag{10b}$$

Initial nonzero entries of matrix $[\tilde{K}]$ are given in Eq. (13) identified from linear behavior of the system at very low-level excitations

Now by inserting expressions developed in Eqs. (9a), (9b) and (10a), (10b) into Eqs. (3a) and (3b) one may determine the changes in the stiffness parameters from this set of equations. Rearranging Eqs. (3a) and (3b), a product of the nonlinear normal mode is obtained

$$\{\tilde{U}(\Omega)\} = \frac{\{\tilde{\phi}_s\}^T\{P\}\{\tilde{\phi}_s\}}{\tilde{\omega}_s^2(1+i\eta_s^2)-\Omega^2} = \{U(\Omega)\} - \sum_{\substack{r=1\\ r\neq s}}^{N}\frac{\{\phi_r\}^T\{P\}\{\phi_r\}}{\omega_r^2-\Omega^2} \tag{11}$$

The vector $\{\tilde{U}(\Omega)\}$ is known and is determined by subtracting the effects of remaining linear modes from the measured frequency responses. Inserting the expression obtained in Eqs. (9a), (9b) and (10a), (10b) for the nonlinear modal mode into Eq. (11), one obtains

$$(\omega_s^2 + \lfloor J_\omega \rfloor\{\Delta k\} - \Omega^2)\{\tilde{U}(\Omega)\} - ([J_\phi]\{\Delta k\} + \{\phi_s\})^T\{P\}([J_\phi]\{\Delta k\} + \{\phi_s\}) = 0. \tag{12a}$$

Neglecting the second-order term, this equation can be rearranged as a set of linear equations

$$[(\{\phi_s\}^T\{P\} + \{\phi_s\}\{P\}^T)[J_\phi] - \{\tilde{U}(\Omega)\}\lfloor J_\omega \rfloor]\{\Delta k\} \cong (\omega_s^2 - \Omega^2)\{\tilde{U}(\Omega)\} - \{\phi_s\}^T\{P\}\{\phi_s\} \tag{12b}$$

Iterative solution of set of linear Eq. (12b) results identifications of the stiffness parameters. By inserting these parameters into Eqs. (9a), (9b) and (10a), (10b) the nonlinear normal modes at specified vibration amplitudes are also determined. In each iteration, sensitivity matrices are updated resulting accurate predictions of the stiffness and modal parameters.

There is an invariant relationship between the identified nonlinear normal mode and its contribution in the response q_s. Identifying this relationship, known as an identification method in the nonlinear modal space, one may produce the response to an arbitrary force level using the identified nonlinear normal modes. In the following experimental case study the relations between nonlinear normal mode and its contribution in the response q_s is identified and using this invariant relation nonlinear response at higher excitation force levels are predicted with very good accuracy.

3. Experimental case study

In order to validate the proposed method an experimental case study is conducted. In this case study the dynamic behavior of a fixed-free beam with nonlinear effects at the clamped end are investigated. The test set-up is shown in Fig. 2 where a steel beam is fixed at the base by two bolts and the excitation force is applied at point B and the responses are measured at points A, B and C. The beam properties are $E = 202.403$ MPa, $\rho = 7800\,\text{kg/m}^3$, $L = 0.527$ m and cross-section area of 29 by 4 mm. The bolts are fastened with a torque of 2 N m and their preloads are kept unchanged during the experiments.

The measurements were obtained at different excitation force levels. Initially the structure was excited with a very low amplitude pseudo-random force within frequency range of 0–500 Hz. These measurements were performed to obtain the underlying linear characteristics of the system. To ensure linear behavior of the system at this excitation level the real and imaginary parts of the measured frequency responses were mapped on each other using Hilbert transform. The result was good agreements in the mapping, indicating validity of linear behavior assumption for the system at this excitation force level.

Next a set of single harmonic excitation tests were performed around the second resonance frequency of the system. The force excitations in these measurements were selected at four different levels of 80, 140, 200 and 300 mN. These levels of excitations were high enough to produce nonlinear effects at the clamping support. Identification of the nonlinear effects in the clamped end of beam is performed using the measured responses at force excitations of 80 mN. The nonlinear normal mode corresponding to the second resonance frequency is identified using this set of measurements and is then employed to predict the nonlinear behavior of the system when excited at higher excitation forces. Predictions of the response using the identified nonlinear normal mode at force excitation levels of 140, 200 and 300 mN match the experimental observations with very good precisions. This indicates success of developed strategy in identifying physically meaningful models for the nonlinear effects in the boundary. The following reports details of the identification procedure.

3.1. Identification of the linear model

A finite element model for the beam is developed using 20 Euler–Bernoulli 2 noded/4° of freedom beam elements. The accelerometer and force transducer added masses are introduced to the mass matrix of the model. This reduces the uncertainty in the model to the parameters of the clamping joint. The stiffness of the clamped end of the beam is modeled using a two by two stiffness matrix. The three parameters of this matrix namely the lateral stiffness, the bending stiffness and the coupling stiffness term are identified by matching the first four observed modes of the structure. These linear

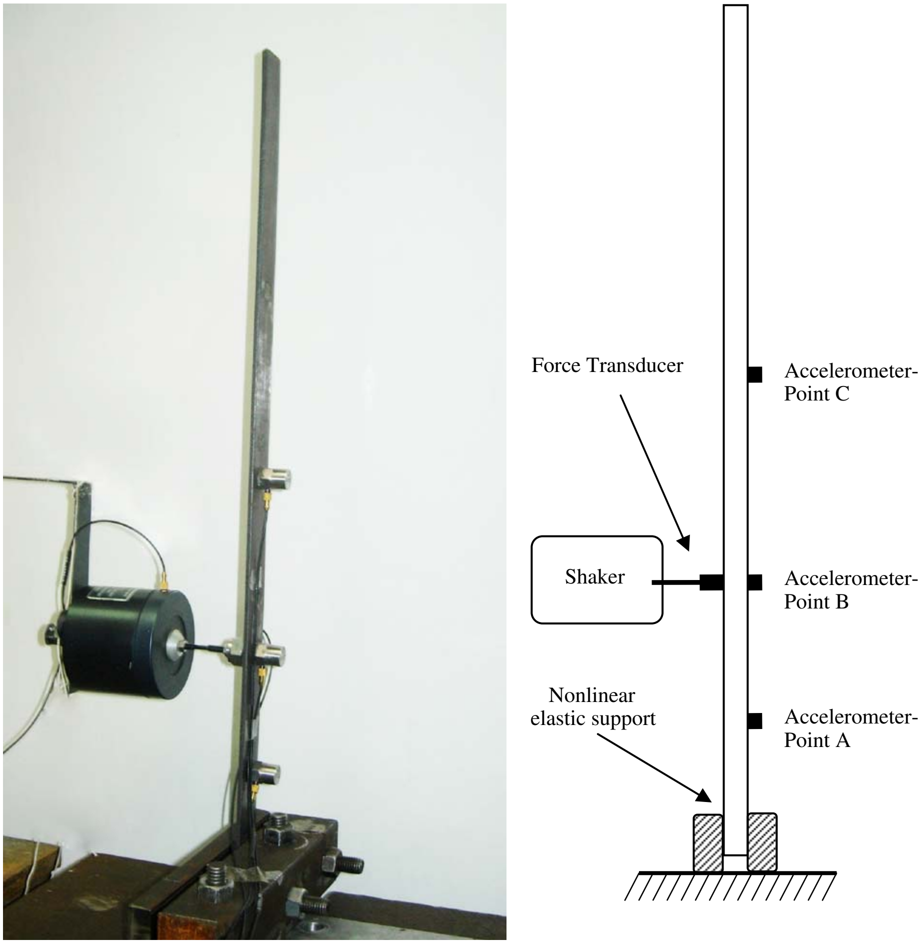

Fig. 2. The test set-up.

Table 1
Identified linear modes

Mode # i	$\omega_i/2\pi$ (Hz)	η_i^2
1	12.1872	0.0030
2	76.9668	0.0022
3	213.9996	0.0017
4	407.0269	0.0021

modes are obtained with a low amplitude random force excitation in the frequency range of 0–500 Hz and are tabulated in Table 1.

The linear boundary parameters are obtained by solution of the structure characteristic equations as proposed by Ahmadian et al. [3]. Characteristic equations of the linear system can be formed using the four identified modes with the boundary stiffness parameters as unknowns. There are four nonlinear complex equations with three complex-valued parameters to be identified. Solution of these characteristic equations results the following stiffness matrix for the clamped boundary:

$$K = \begin{bmatrix} 934396.4234 + 4456.5792 \times i & 16955.2225 + 588.4493 \times i \\ 16955.2225 + 588.4493 \times i & 2055.9615 + 42.5173 \times i \end{bmatrix} \quad (13)$$

Fig. 3 compares the predictions of the identified model with the experimental results. Analytical model predicts the frequency response of the structure at excitation point (point B) with high accuracy.

An accurate linear model for the boundary of structure is created in this section. In the followings variations of the boundary model due to nonlinear effects at higher loadings are identified.

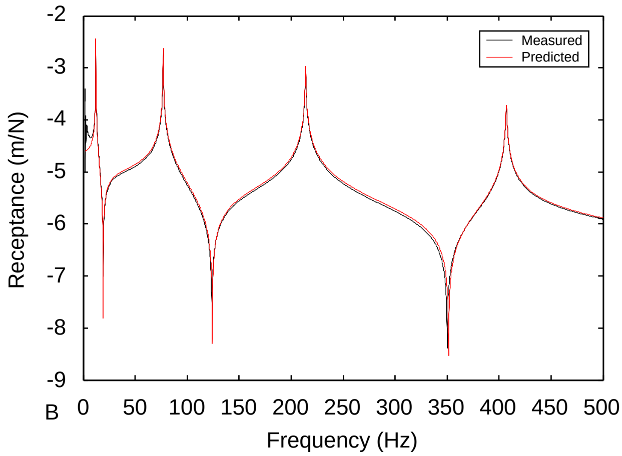

Fig. 3. Measured and predicted linear receptance at point B.

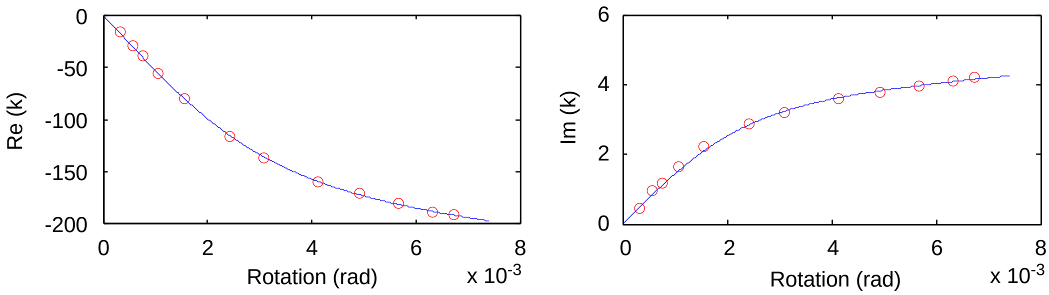

Fig. 4. Variations of bending stiffness versus $|q_2|$ (identified—dots and curve fitted—solid line).

3.2. Identification of the nonlinear model

Investigation on nonlinear behavior of the clamped end was performed by exciting the structure with a single harmonic force in vicinity of the second resonance frequency. The amplitude of excitation was set to 80 mN and consequently at this level of excitation the softening effects in the measured frequency response functions are observed (see Fig. 7). The nonlinear measured frequency responses at three locations of A, B, and C on the beam are used to construct the nonlinear normal mode of the structure at its second resonance frequency.

The nonlinear behavior in the clamped end is governed by the micro-slips between clamping surfaces. One of the most important properties of such frictional connections is the relationship between loading and deformation, particularly in the tangential direction. The friction forces produced between connecting parts are the primary source of damping in the structure and cause localized nonlinear behavior. Friction forces are displacement dependent and any model selected to represent the restoring forces in the boundary must resemble two major contributions of shear deformations in the clamping mechanism, i.e. the softening of the clamping stiffness and saturation of its shear force at the slip. Base on this physics involved in the clamped joint and sensitivity analysis of the second mode to the joint parameters, the bending stiffness parameter was selected as the sensitive parameter and the other two parameters were kept unchanged.

The changes in the bending stiffness parameter at different deformation amplitudes are identified using Eqs. (12a) and (12b). It should be noted the excitation force level is kept constant at 80 mN and variations in the response level is due to changes in the excitation frequency in vicinity of the second resonance frequency. At each response amplitude level the nonlinear normal mode is also identified. The identified changes in nonlinear bending stiffness, nonlinear eigenvalue, and nonlinear normal mode are shown in Figs. 4–6, respectively.

Capturing the invariant relations between nonlinear normal mode and amplitude of its contribution in the response, i.e. q_s, enables one to predict the behavior of the nonlinear structure at different loading conditions. This is true provided that only the considered mode acts in a nonlinear manner in the response and other modes behave linearly. To identify this invariant relationship, the deviations of nonlinear normal mode from its initial linear counterpart are expressed using a hyperbolic function with the parameter q_s as its independent variable, i.e.

$$\Delta\tilde{\omega}_2^2(|q_2|) = (\beta_1 + \beta_2|q_2|)\tanh(\beta_3|q_2|) \tag{14a}$$

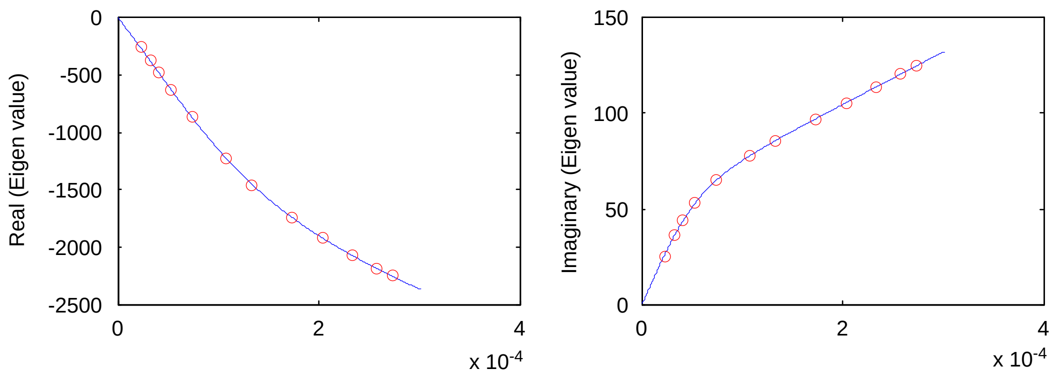

Fig. 5. Variations of nonlinear eigenvalue versus $|q_2|$ (identified—dots and curve fitted—solid line).

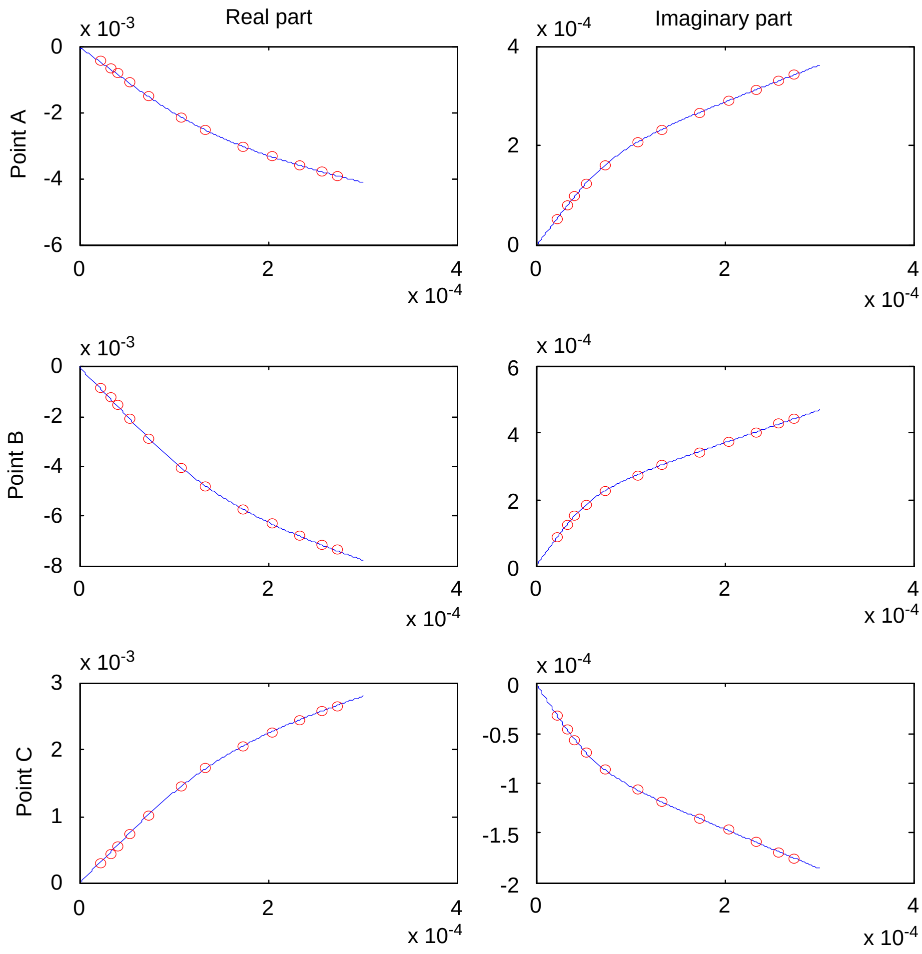

Fig. 6. Variations of nonlinear normal mode versus $|q_2|$ (identified (dots)—curve fitted (solid line)).

$$\Delta\tilde{\phi}_{i2}(|q_2|) = (\gamma_{i1} + \gamma_{i2}|q_2|)\tanh(\gamma_{i3}|q_2|), \quad i = 1, 2, 3 \tag{14b}$$

where q_2 is the contribution of the nonlinear normal mode in the response and $\beta_j, \gamma_{ij}, j = 1, 2, 3$ are constant coefficients to be determined from curve fitting procedure. The results of fitted function are plotted in Figs. 4–6. They successfully regenerate the identified nonlinear normal modes and can be used to define the invariant relations between the second nonlinear normal mode and q_2.

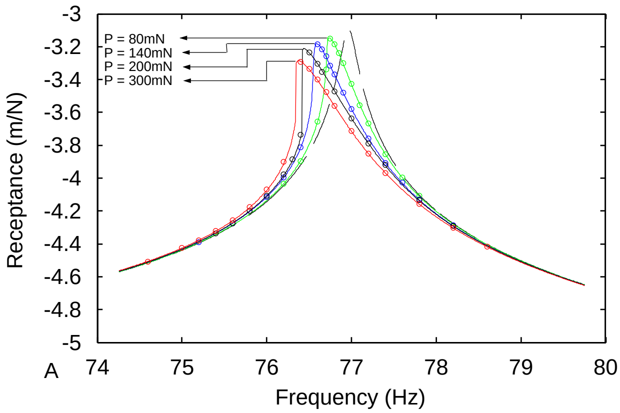

Fig. 7. Linear (dashed line) and nonlinear (predicted—solid lines, measured—dots) receptances at point A.

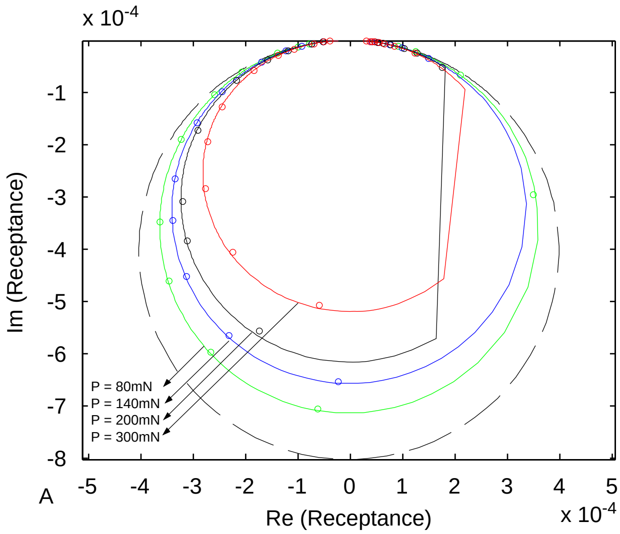

Fig. 8. Nyquist plots of receptances at point A.

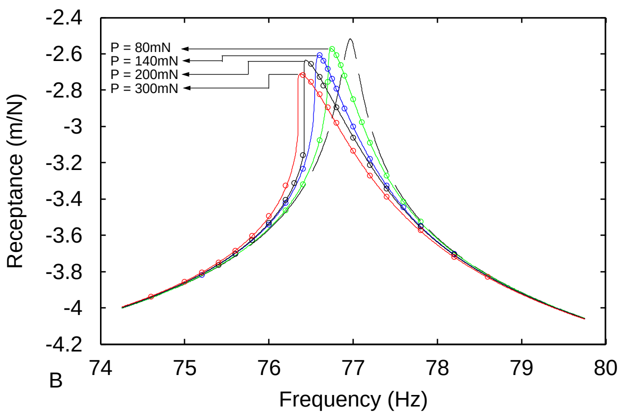

Fig. 9. Linear and nonlinear receptances at point B.

Identifying nonlinear parameters in modal space from the responses at excitation level of 80 mN, the nonlinear response to higher levels of excitations namely 140, 200, 300 mN are predicted using identified nonlinear normal model and are compared with the corresponding experimental observations. The results are shown in Figs. 7–12. The predicted results

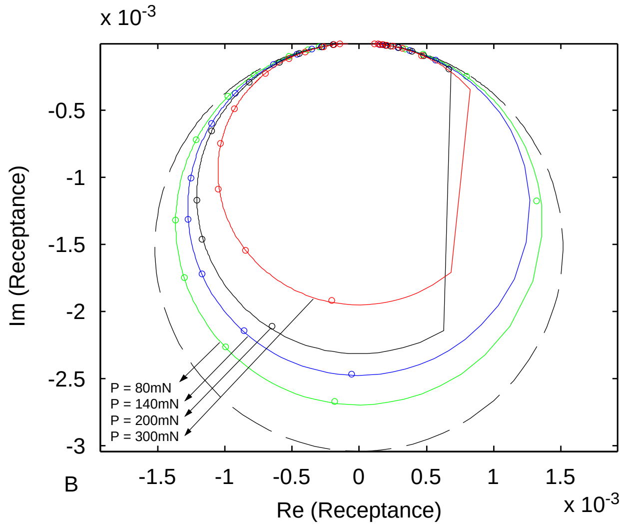

Fig. 10. Nyquist plots of receptances at point B.

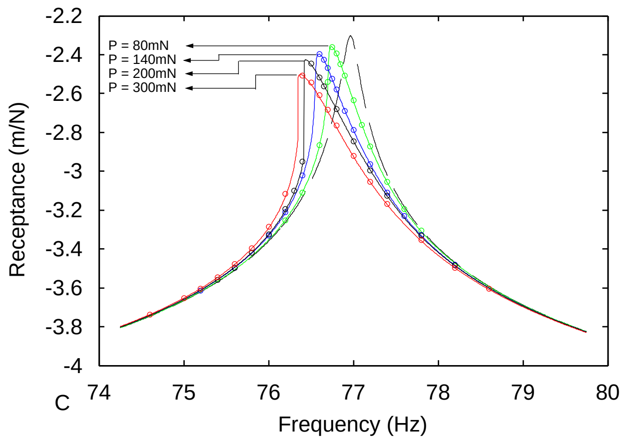

Fig. 11. Linear and nonlinear receptances at point C.

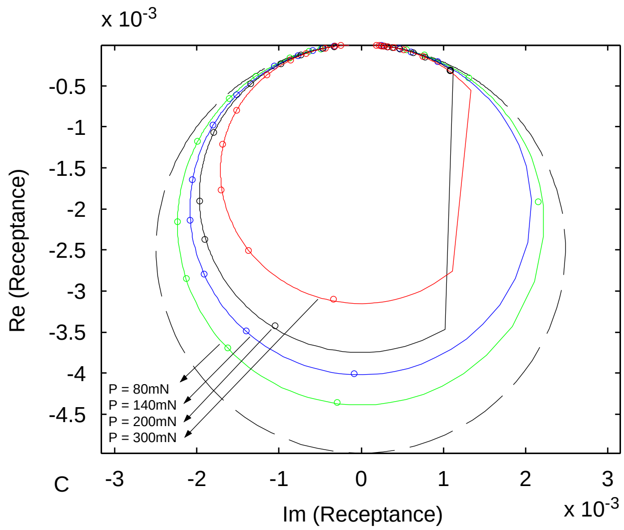

Fig. 12. . Nyquist plots of receptances at point C.

show very good agreements with the experimental observations. In these figures, linear receptances are plotted using dashed line and solid lines show the nonlinear responses at different excitation levels.

The success in representing a model capable of predicting nonlinear response of the structure is because of identifying the true invariant relations between the nonlinear normal mode and its participation coefficient in the response. Accurate identification of these invariant relations was made possible by defining them using physical parameters of the boundary stiffness.

4. Conclusion

Response of a structure with local nonlinearities is presented using a nonlinear normal mode. There is an invariant relation between the nonlinear mode parameters and its corresponding generalized coordinate amplitude in which by identifying this relation structure responses can be predicted at different excitation levels. This paper proposes a new method in defining this invariant relation which makes use of analyst intuitions on physical phenomenon involve in the nonlinearity mechanism. In conventional methods of reconstructing a nonlinear normal mode, the mode variations due to change in the response amplitude is defined as a linear combination of some linear modes. This makes the identification procedure very sensitive to the number and range of selected modes. In the proposed method variations of nonlinear normal modes are related to the physical parameters involved in the nonlinear local effects. This employs the physical relations of the nonlinear mechanism and reduces the parameters required to define the variations of the nonlinear normal mode significantly. The efficiency of the method is demonstrated using an experimental case study in which an excited nonlinear normal mode of structure is successfully identified. The success of proposed identification procedure is demonstrated by accurately predicting response of the structure in higher levels of excitation forces.

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